IA841 – Modelagem de Sólidos

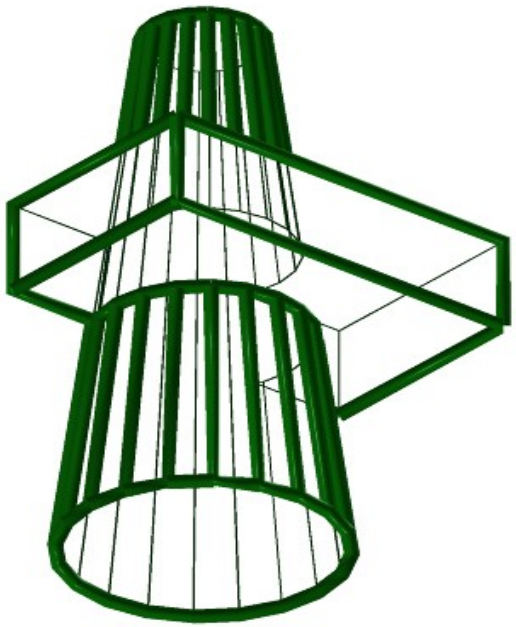
Operações Booleanas em B-rep

Hoffmann: Capítulo 3

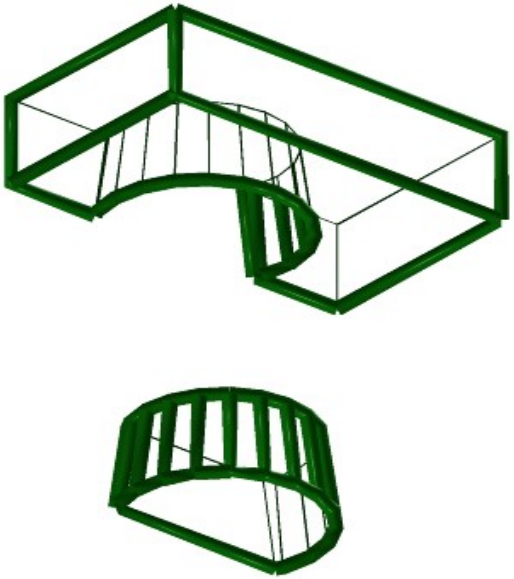
Operações Booleanas

Como implementá-las em BRep?

$A - B$



$A \cup B$

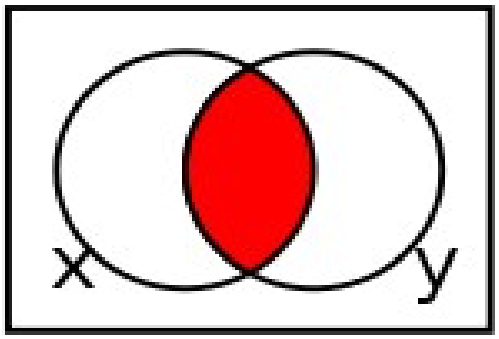


$A \cap B$

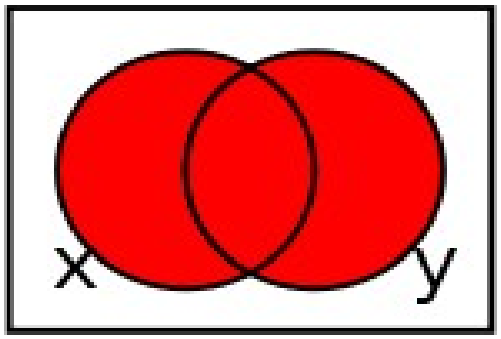
$B - A$

http://www.cs.technion.ac.il/~irit/user_man.html

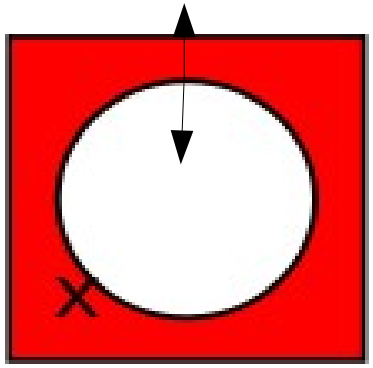
Complemento, União e Diferença → Interseção



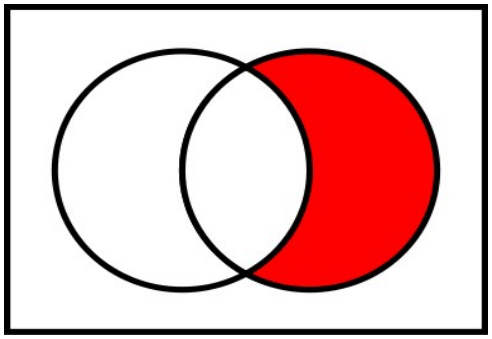
$x \wedge y$



$X \vee Y = \neg(\neg X \wedge \neg Y)$

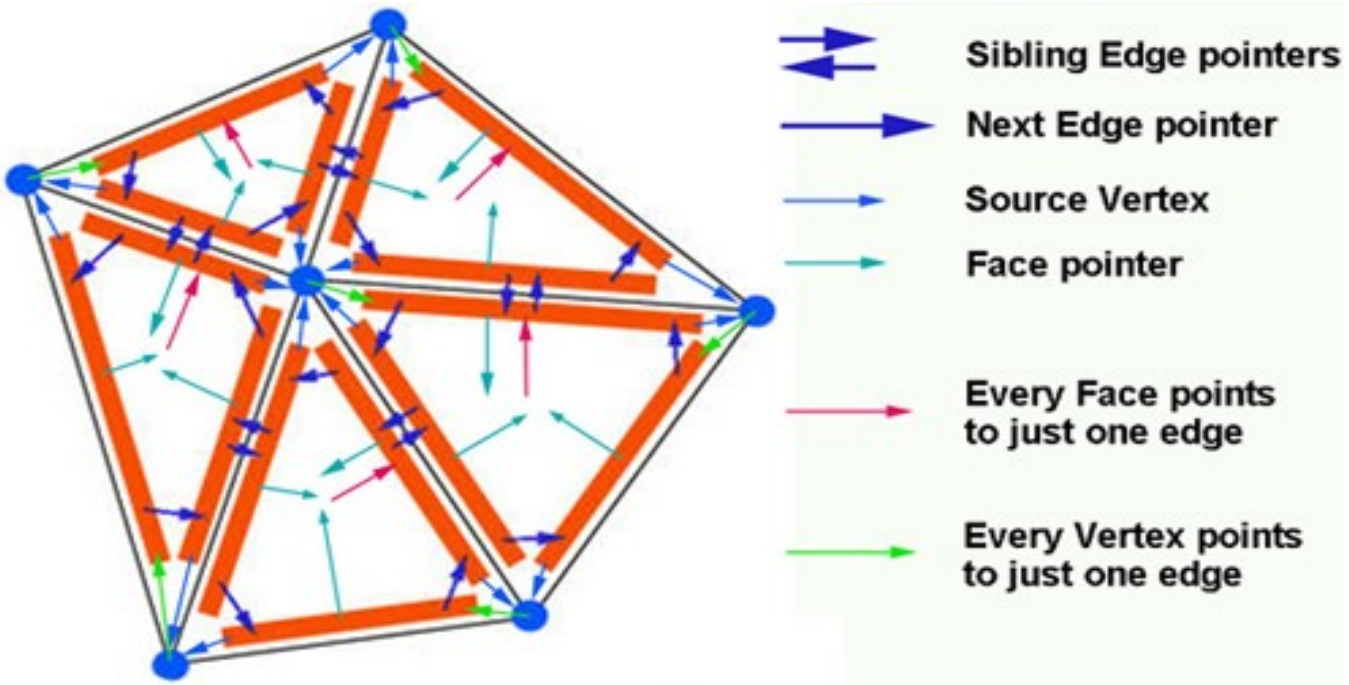


$\neg x$



$Y - X = Y \wedge \neg X$

Informações Topológicas



Vértices: arestas e faces adjacentes
Arestas: vértices e faces adjacentes
Faces: sequências fechadas de vértices e arestas (CW)

Informações Geométricas Irredundantes

- Vértices (x,y,z,w)
- Plano que contém faces

$$n_x x + n_y y + n_z z + d = 0$$

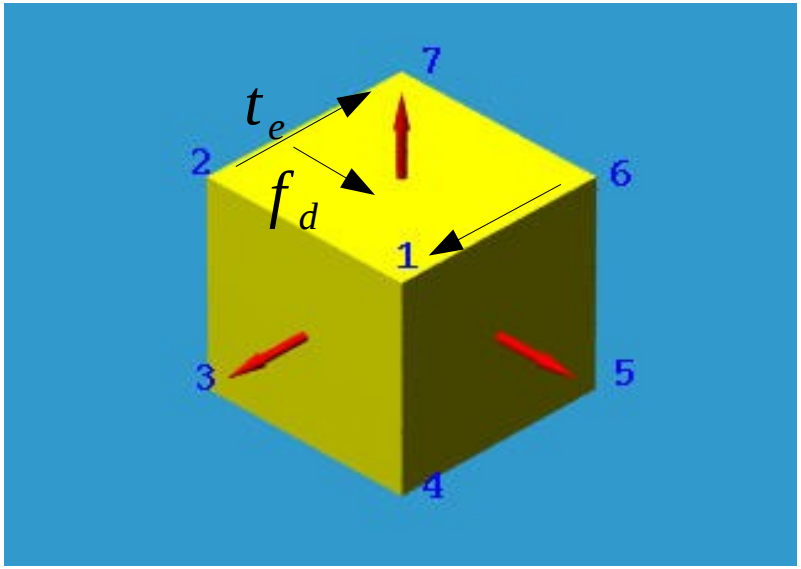
- Vetor normal da face

$$(n_x, n_y, n_z, 0)$$

- Vetor de direção da face

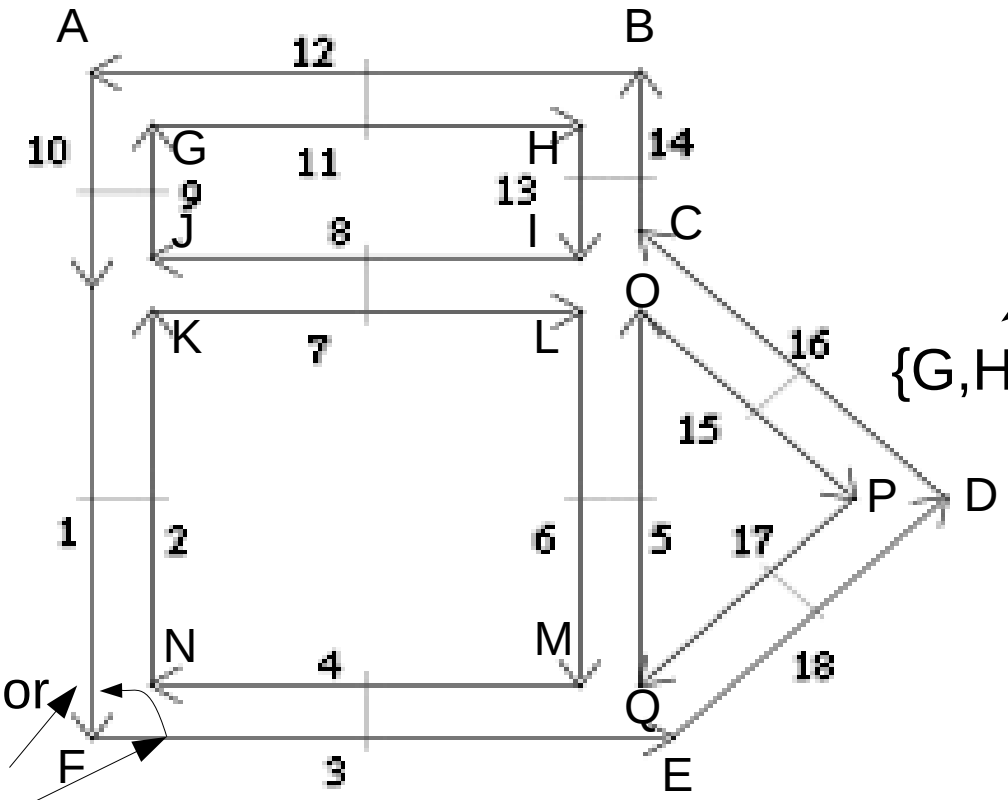
$$f_d = t_e \times \vec{n}$$

- e outros dados



Faces Válidas

- Sequências fechadas de vértices/arestas delimitam regiões de áreas não nulas.



Par de arestas delimitador de um setor

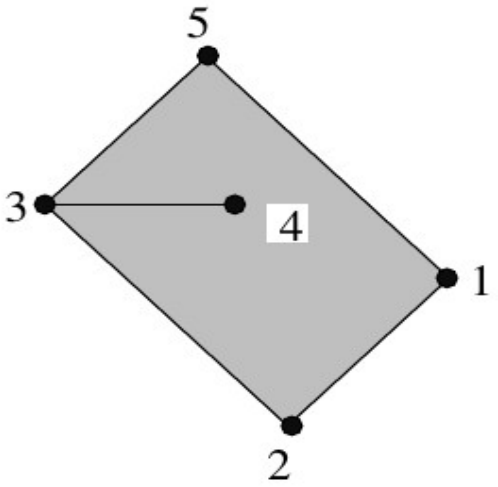
{A,B,C,D,E,F}

{G,H,I,J}

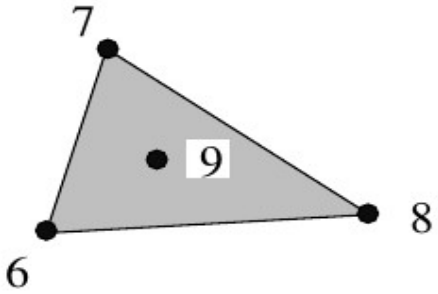
{K,L,M,N}

{O.P.Q}

Faces com ciclos degenerados

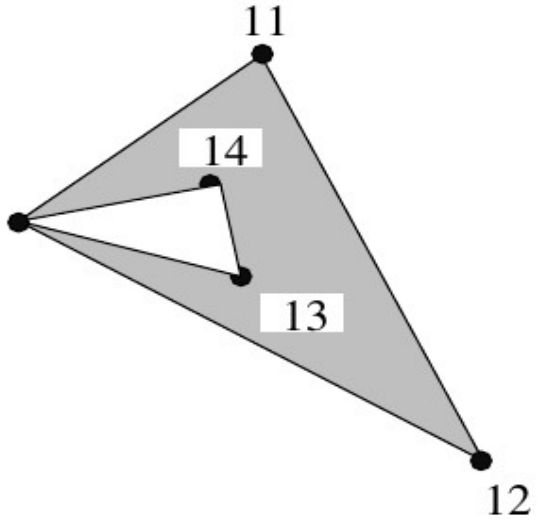


{1,2,3,4,3,5}



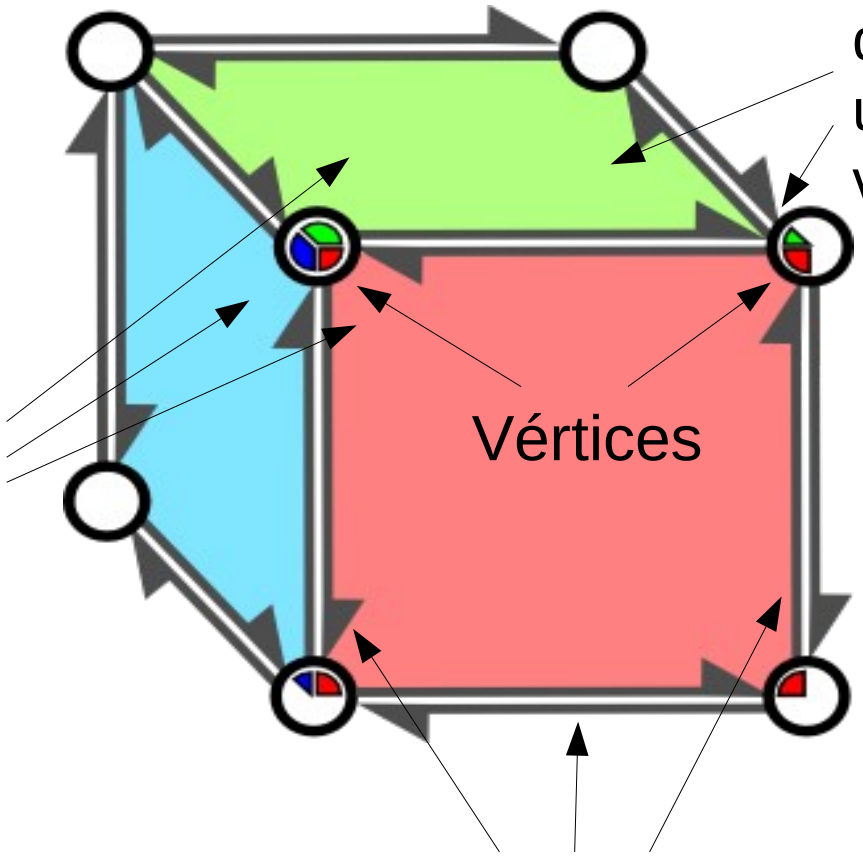
{6,7,8}

{9}

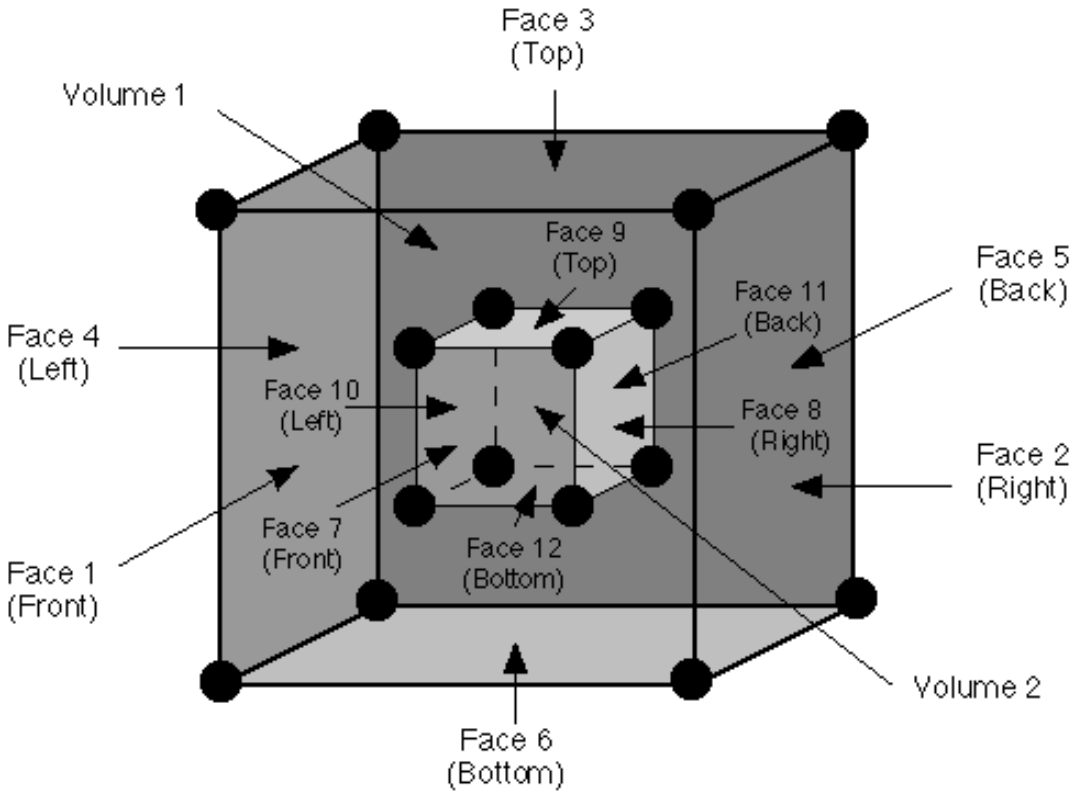


{12,10,13,14,10,11}

Arestas e Vértices



Cascas (*Shells*)



Interseção Casca/Casca

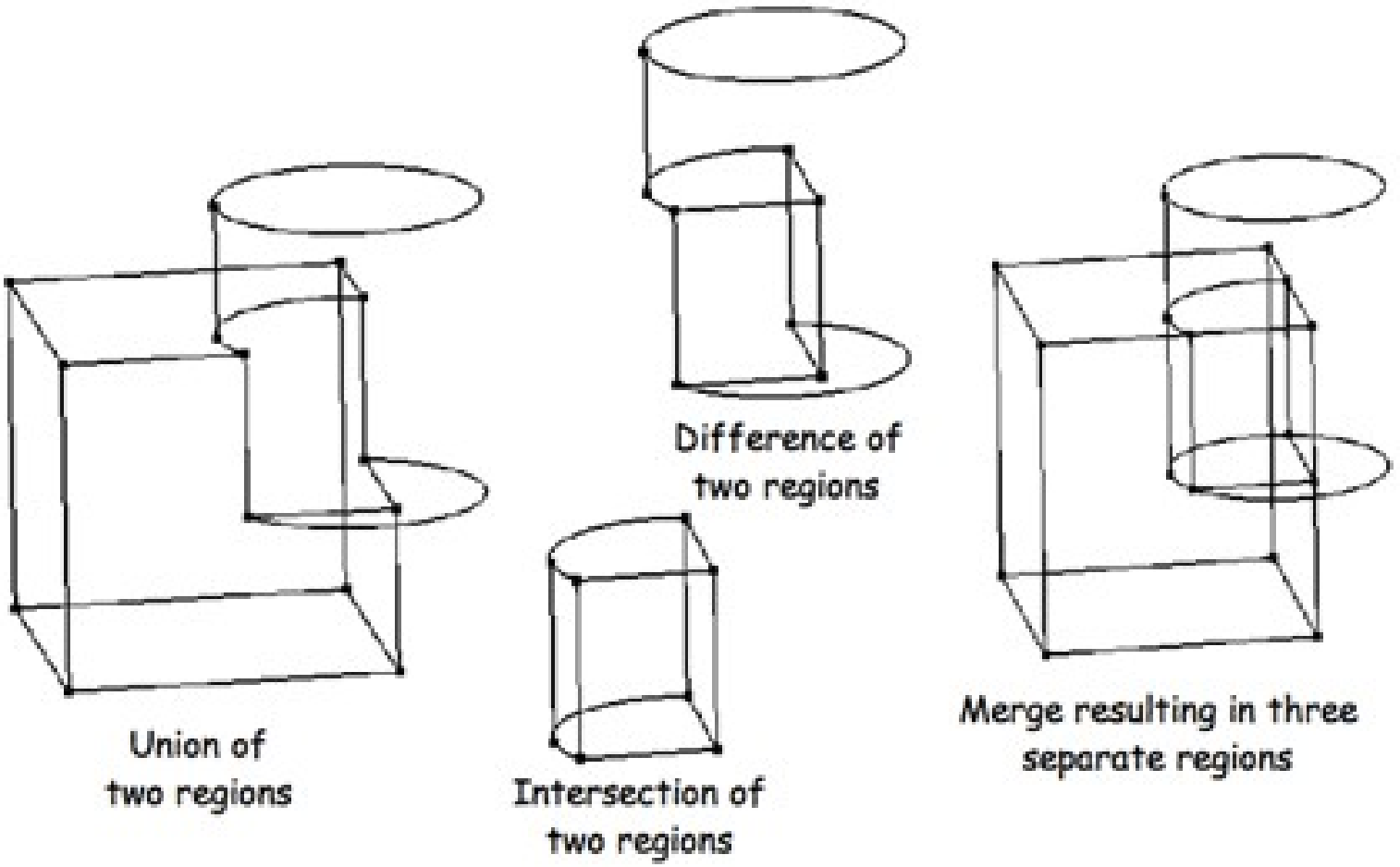
1. Determine which pairs of faces $f \in A$ and $g \in B$ intersect. If there are none, test shell containment only and skip steps 2 through 4.

2. For each face f of A that intersects a face of B, construct the cross-section of B with the plane containing f. Then determine the surface area of $A \cap^* B$ that is contained in f.

3. By transferring the relevant line segments discovered in step 2, determine the faces of B that contain some of the surface area of $A \cap^* B$ and must be subdivided. Subdivide these faces, and by exploring the face adjacencies of B, find and add all those faces of B contained in the interior of A. Likewise, find and addall faces of A contained in the interior of B.

4. Assemble all faces so found into the solid $A \cap^* B$.

Propagação das Decisões



Um Algoritmo Robusto

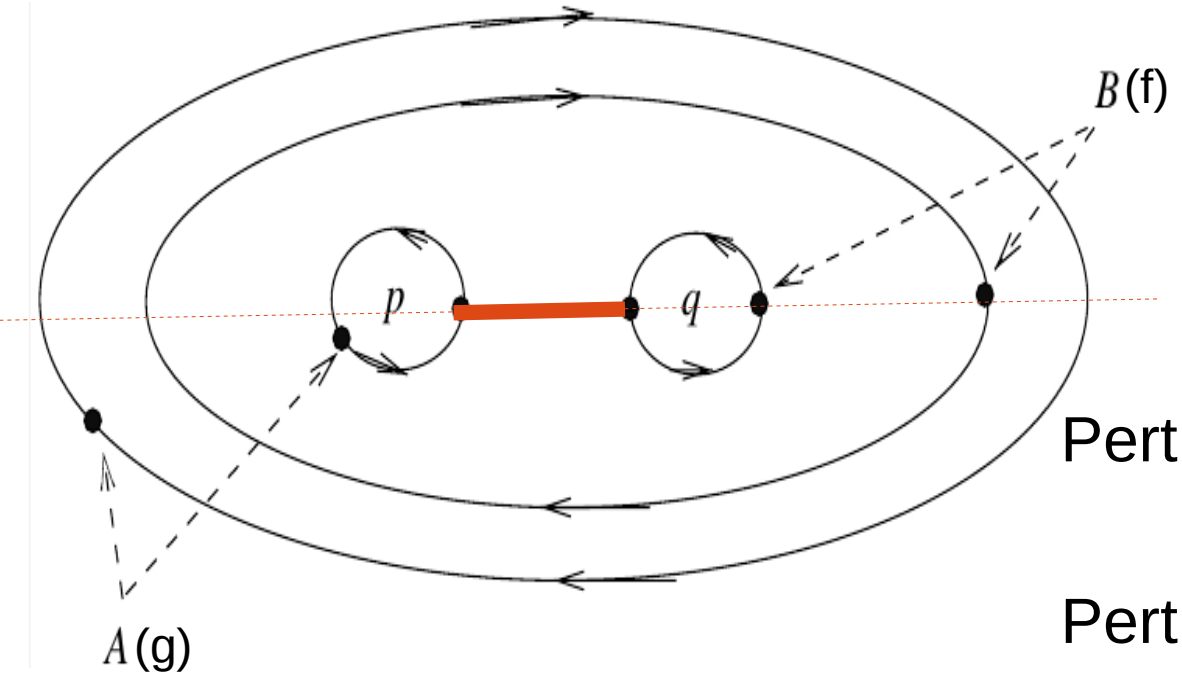
1. Determine which pairs of faces $f \in A$ and $g \in B$ intersect. If there are none, then do a shell-containment test only and skip steps 2 through 4.

2. For each intersecting pair of faces, f of A and g of B, construct the points and curves in which they intersect. For each intersection, analyze the three-dimensional neighborhood and transfer its elements to all adjacent faces of A and of B.

3. By exploring the face adjacencies of A and of B, find and add all those faces of either solid that are in the interior of the other.

4. Assemble all faces into the solid $A \cap^* B$.

Posição Relativa entre 2 Cascas/Faces Coplanares



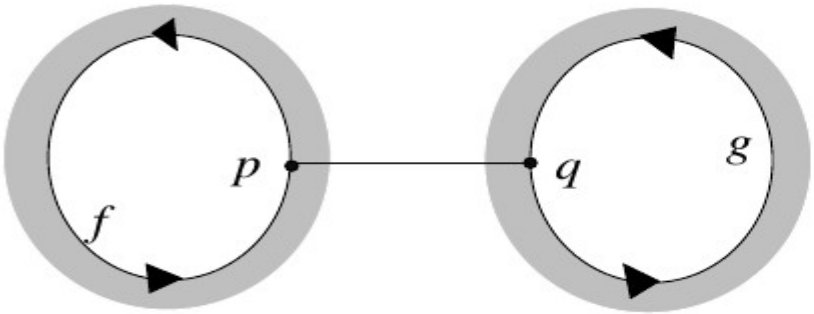
Pertinência Segmento/Face

↕

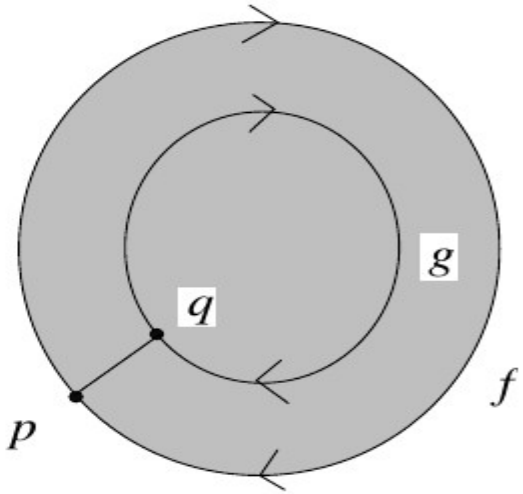
Pertinência Segmento/Sólido

Test	Example	Action
in f, in g	f and g intersect	both components are kept
in f, out g	g is contained in f	the g component is kept
out f, in g	f is contained in g	the f component is kept
out f, out g	f and g do not intersect	neither component is kept

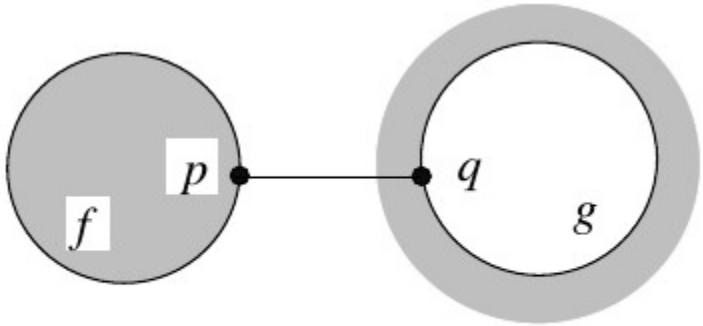
Quatro Casos



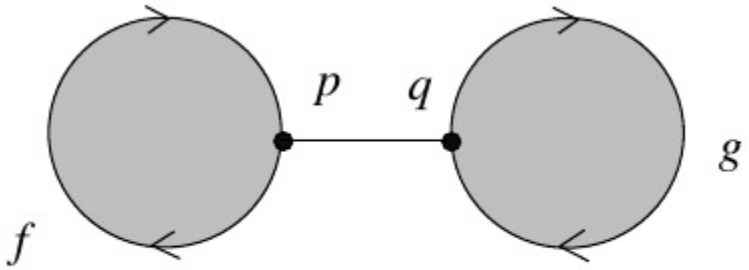
In/In



In/Out



Out/In

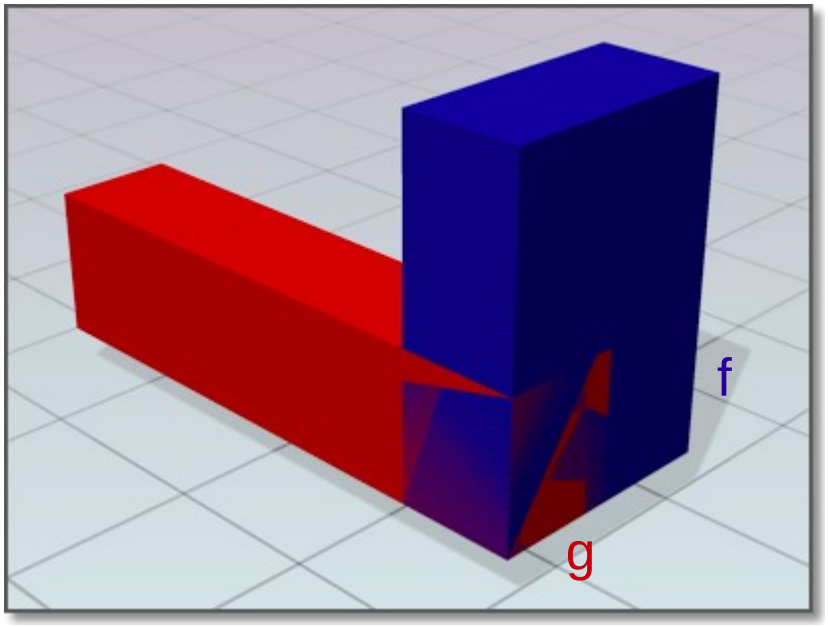


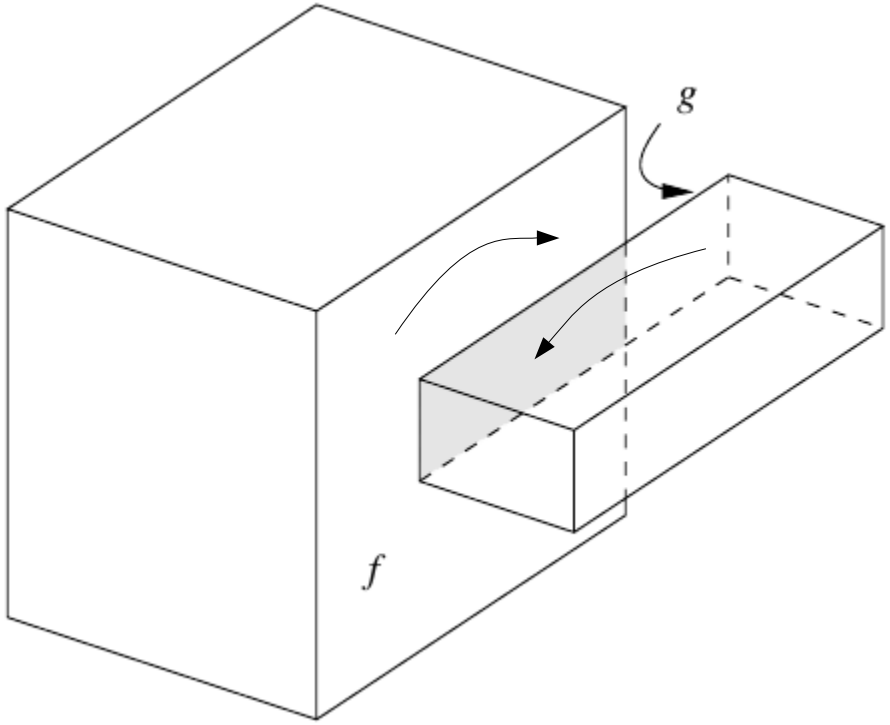
Out/Out

Interseção de Par de Faces

- Interseção face $g \in B$ com o plano $P \supset f$
- Interseção face $f \in A$ com o plano $Q \supset g$
- Agrupamento das intersecões:
    - f, g $\subset$ mesmo plano $\rightarrow$ polígonos
    - P, Q são planos concorrentes $\rightarrow$ pontos e segmentos

Faces Coplanares





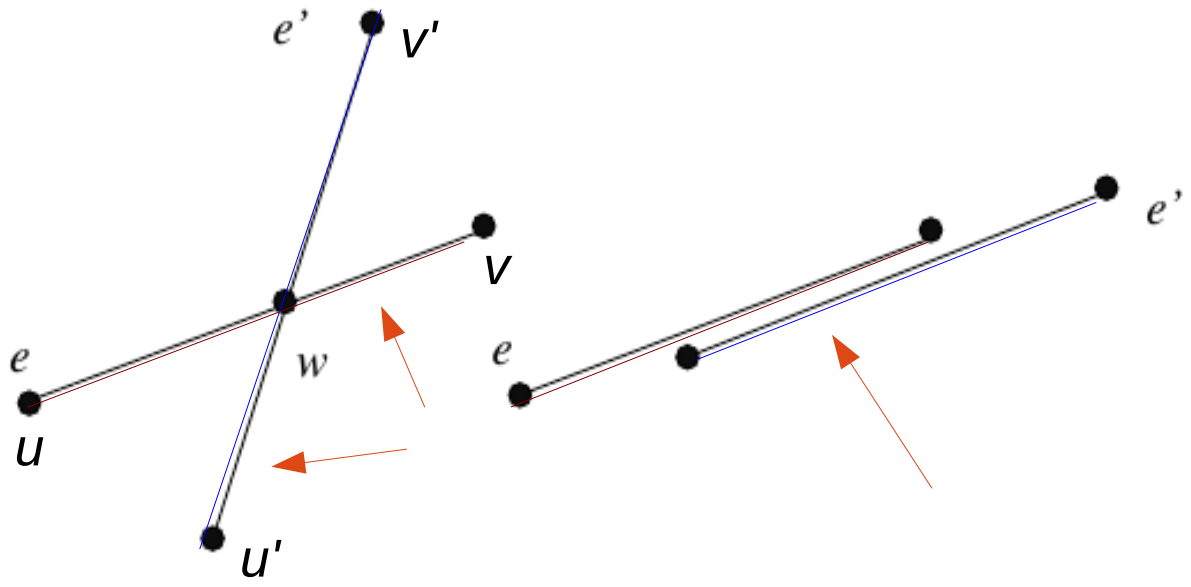
Com orientações coerentes

A ∩* B → A∩B

Com orientações opostas

A ∩* B → discarda

Interseção Aresta/Aresta



Arestas concorrentes

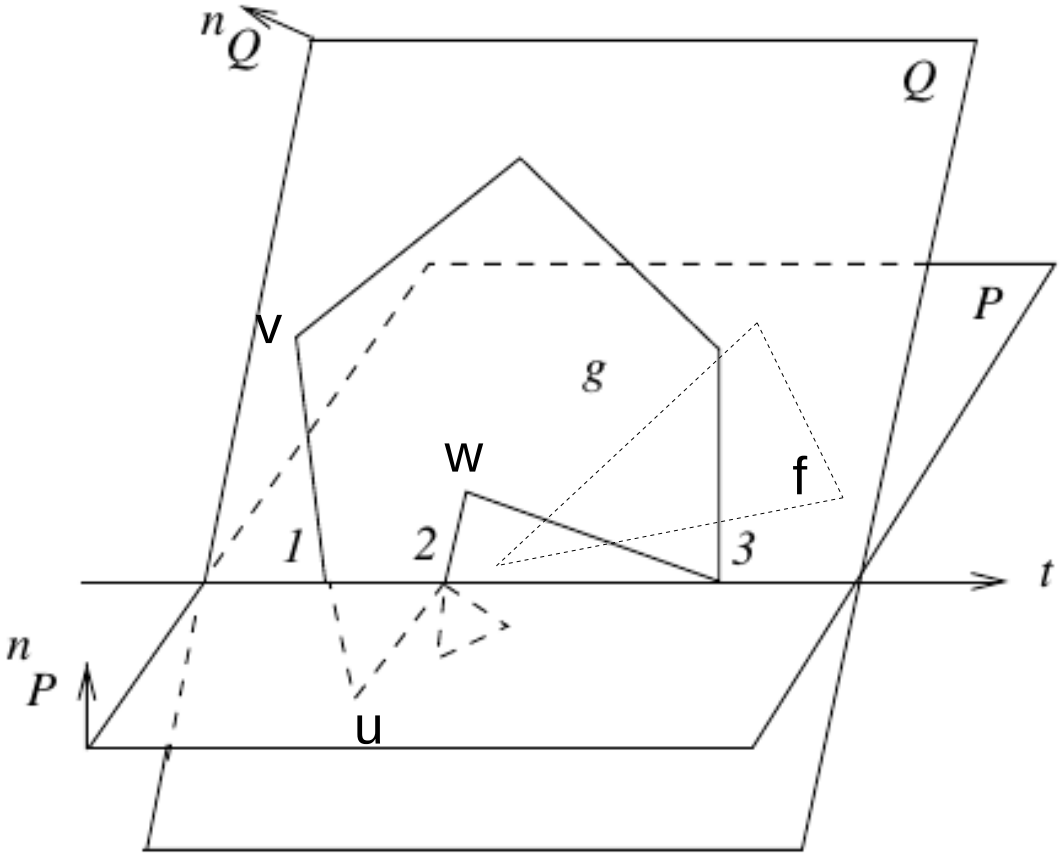
Arestas colineares

$$\left(\overline{u'w}, \overline{wv'} \in g\right) \subset A \; ? \; \overline{u'w}$$

$$\left(\overline{uw}, \overline{wv} \in f\right) \subset B \; ? \; \overline{wv}$$

Determinar intervalos coincidentes

Faces Concorrentes



Interseção Face/Plano

$x_1 < x_2 < x_3$

$\vec{t} \rightarrow x_1, x_2, x_2', x_3, x_3'$

Pareamentos:

v acima e u abaixo de P: x_1

$$x_1\, x_2$$

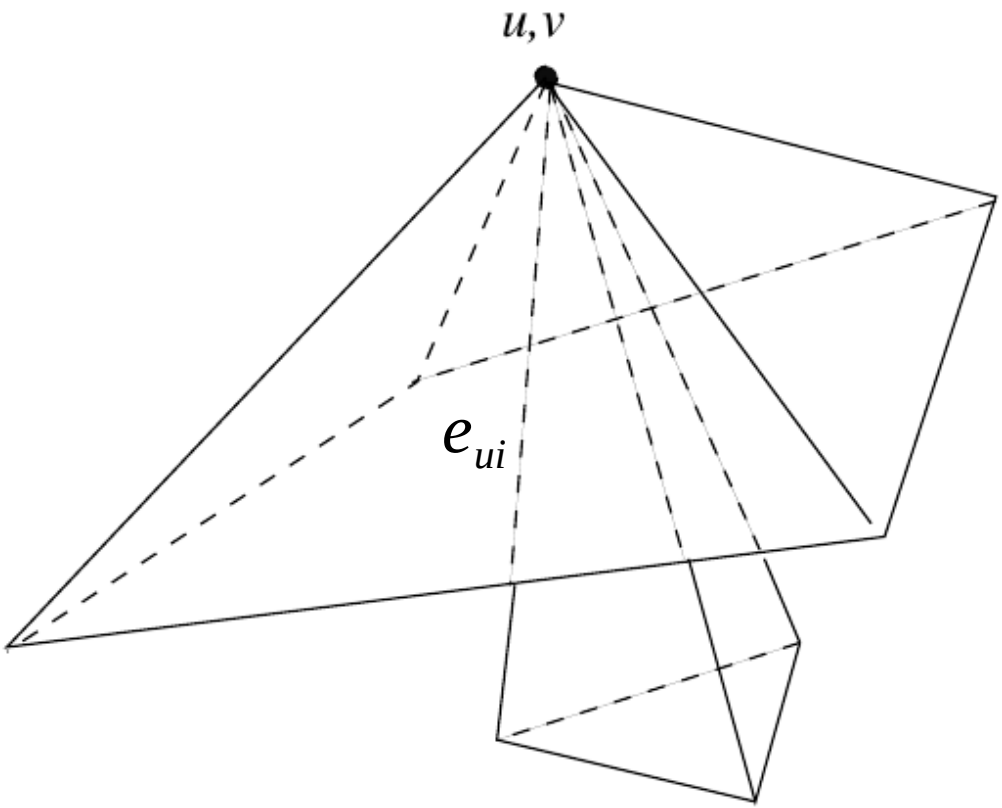
$$-\vec{t} \subset setor : x_2$$

$$x_1\, x_2$$

$$\vec{t}, -\vec{t} \not\subset setor : x_3$$

$$x_3 \quad isolado$$

Interseção Vértice/Vértice



$e_{ui}, u \in A$

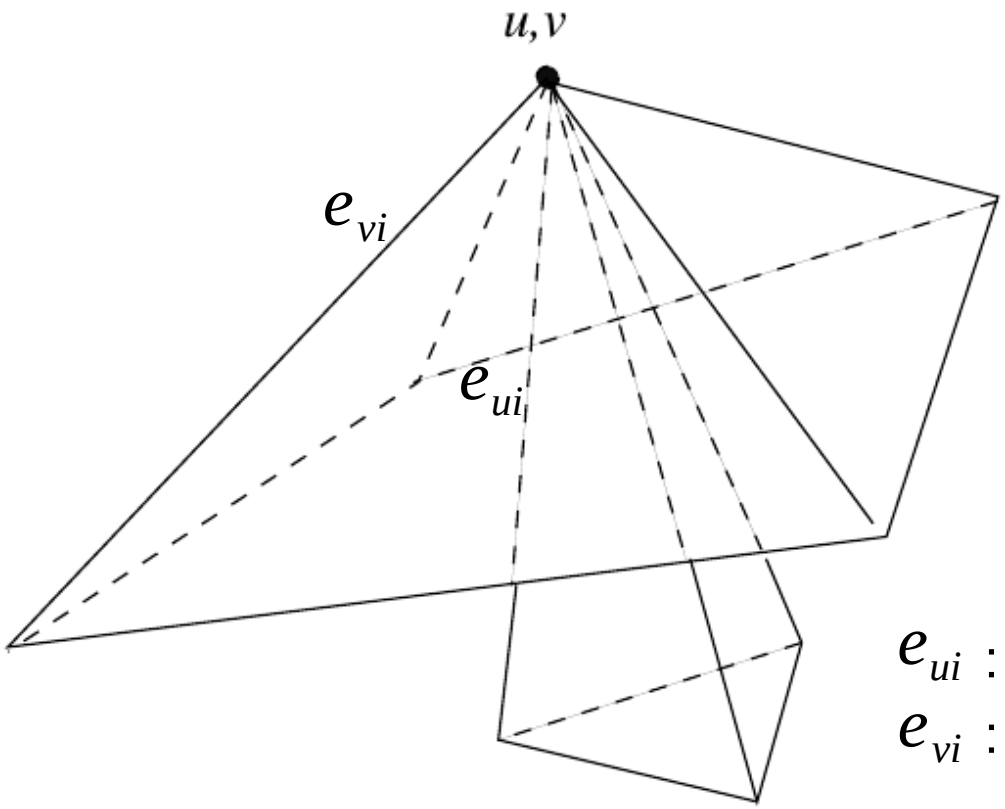
$e_{vi}, v \in B$

Propagação de Adjacências:

$$e_{ui} \subset Cone(v) \rightarrow e_{ui} \in B$$

$$e_{vi} \subset Cone(u) \rightarrow e_{vi} \in A$$

Interseção Vértice/Vértice



$e_{ui}, u \in A$

$e_{vi}, v \in B$

e_{ui} : arestas adjacentes a u

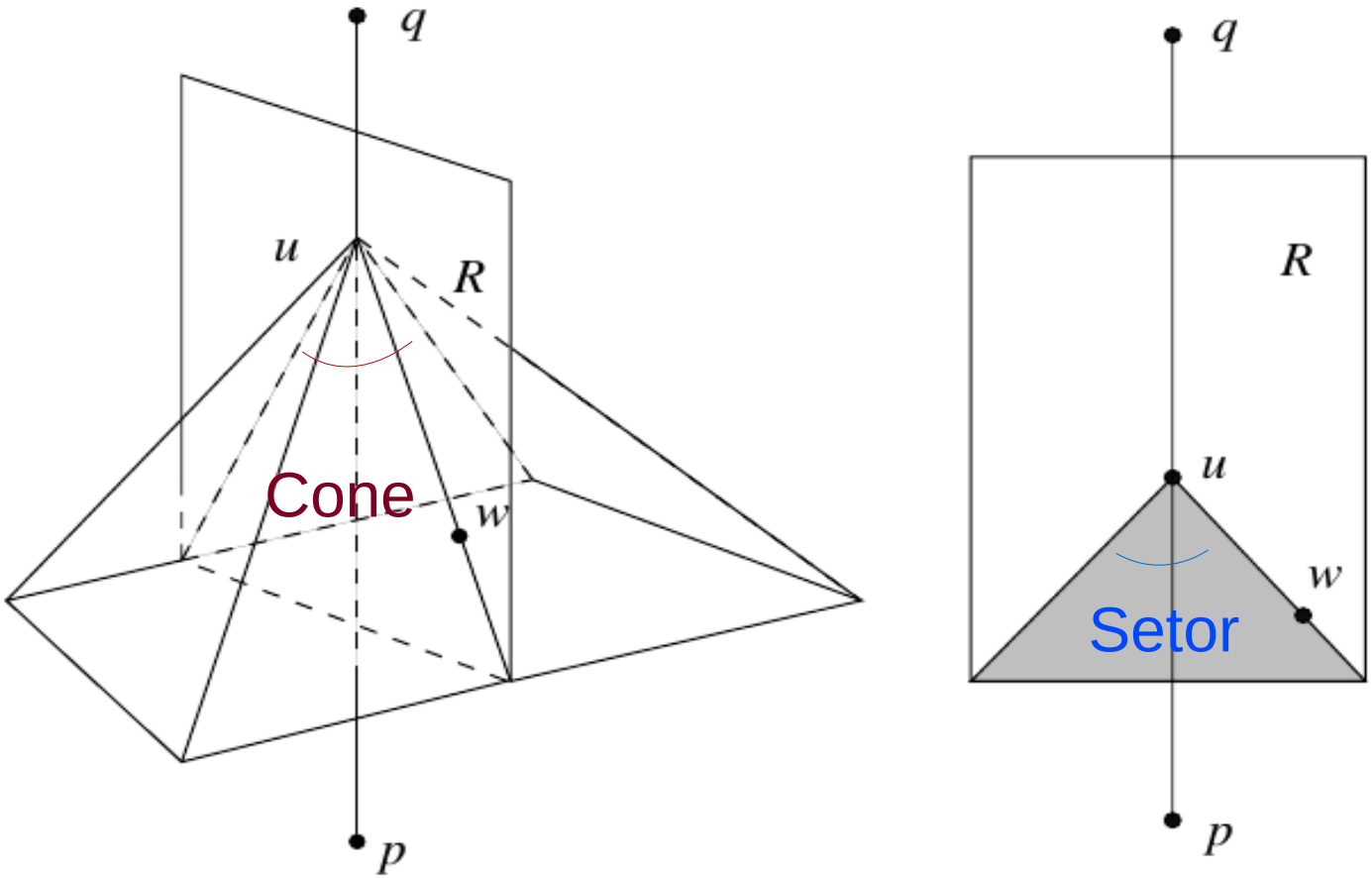
e_{vi} : arestas adjacentes a v

Propagação de Adjacências:

$$e_{ui} \subset Cone(v) \rightarrow e_{ui} \in B$$

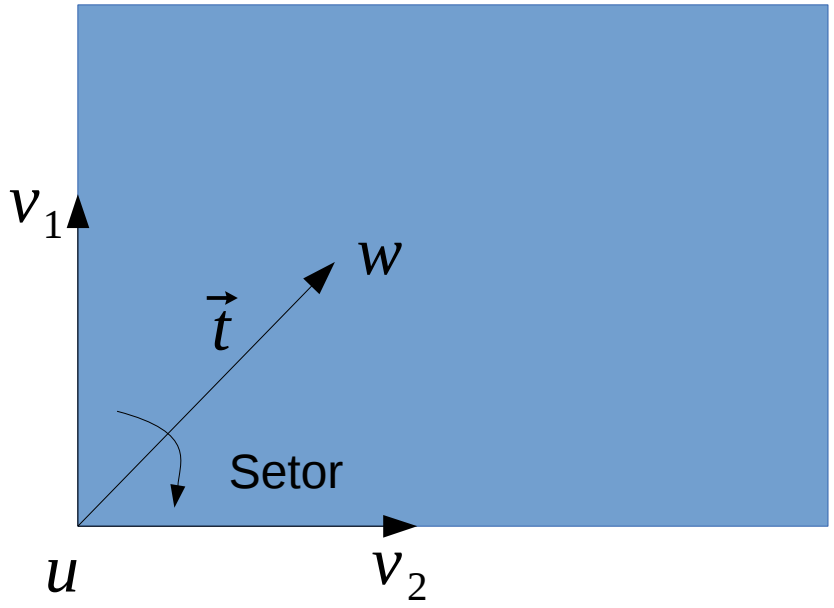
$$e_{vi} \subset Cone(u) \rightarrow e_{vi} \in A$$

Pertinência a um Cone



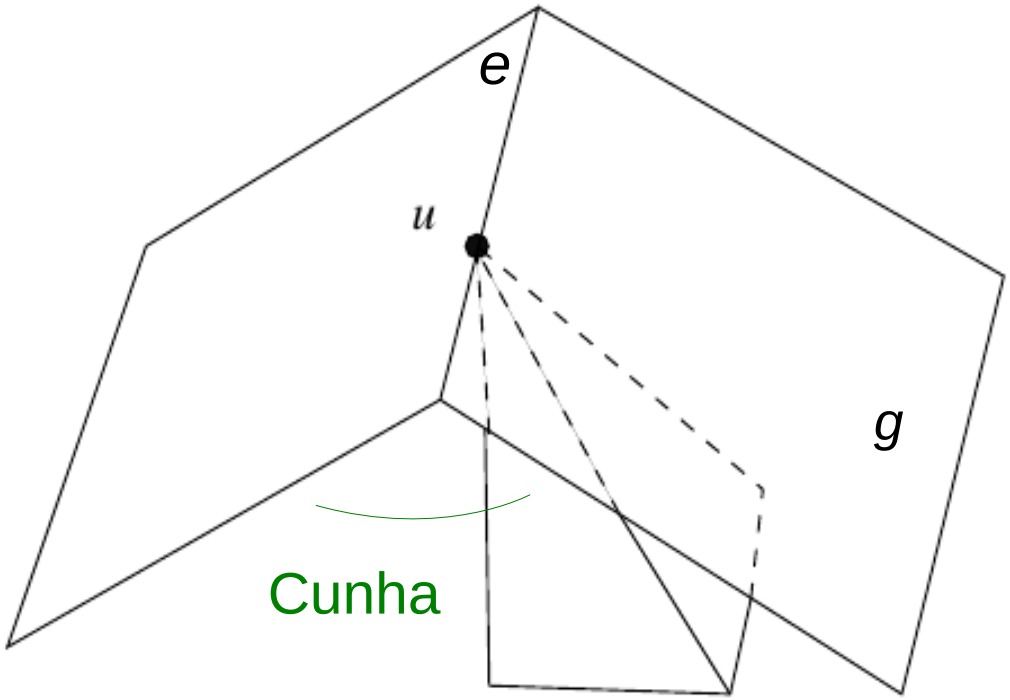
$$\overline{pq} \subset Cone \rightarrow \overline{pq} \subset Setor$$

Pertinência a um Setor



$$sinal\left(\overrightarrow{uv_1}\times\overrightarrow{uw}\right)=sinal\left(\overrightarrow{uw}\times\overrightarrow{uv_2}\right)$$

Interseção Vértice/Aresta

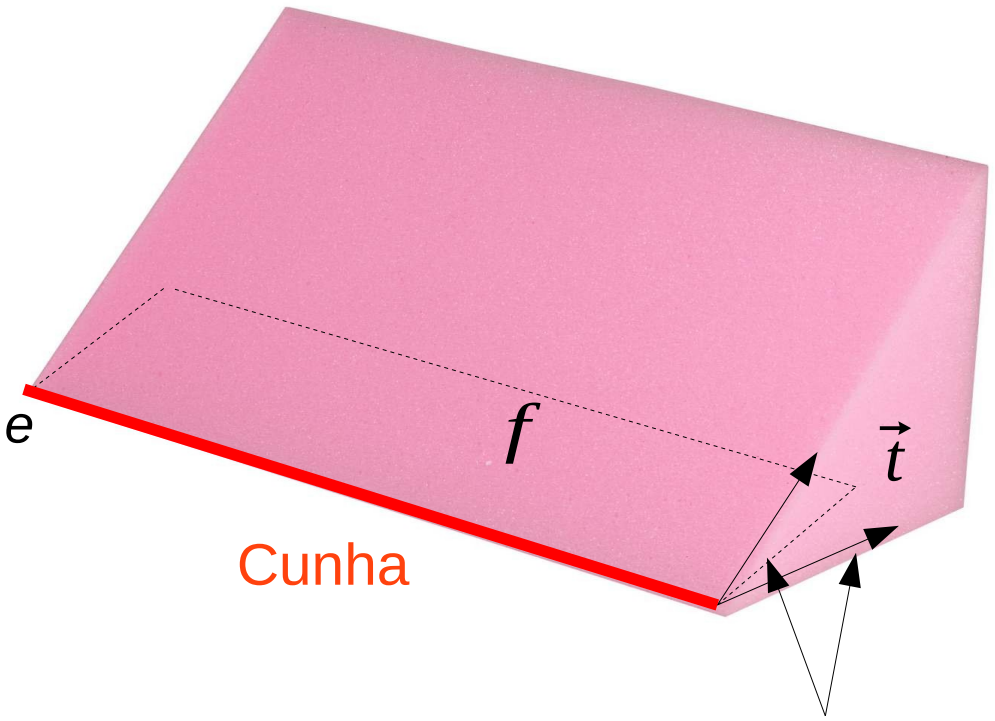


Propagação de Adjacências:

$$e_{ui} \subset Cunha(e) \rightarrow e_{ui} \in B$$

$$\exists e_{vi} \subset Cunha(u) \wedge e_{vi} \not\subset Cunha(u) \rightarrow \exists 2\, arestas \in g$$

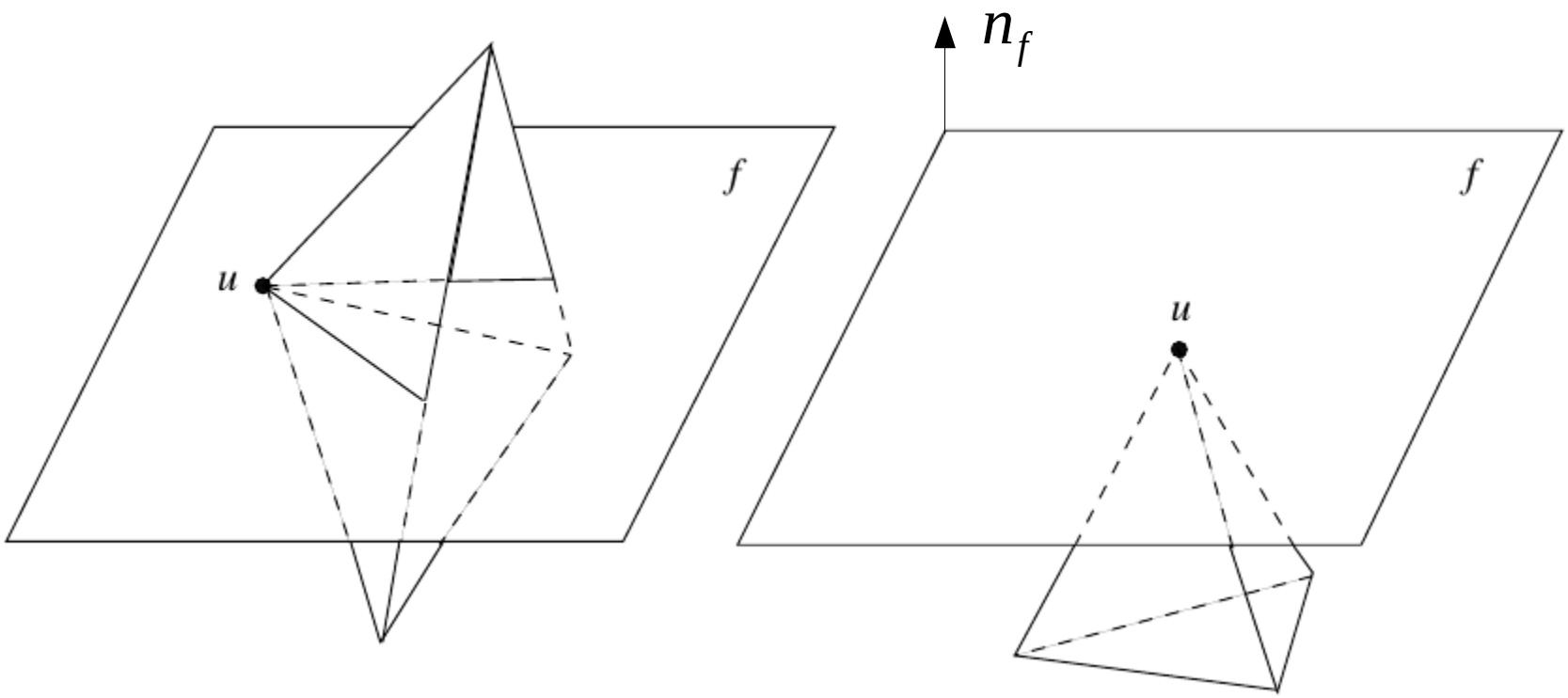
Pertinência a uma Cunha



Vetores de direção de g_i

$$\vec{t} \subset Cunha(e) \rightarrow \vec{t} \subset Setor$$

Interseção Vértice/Face

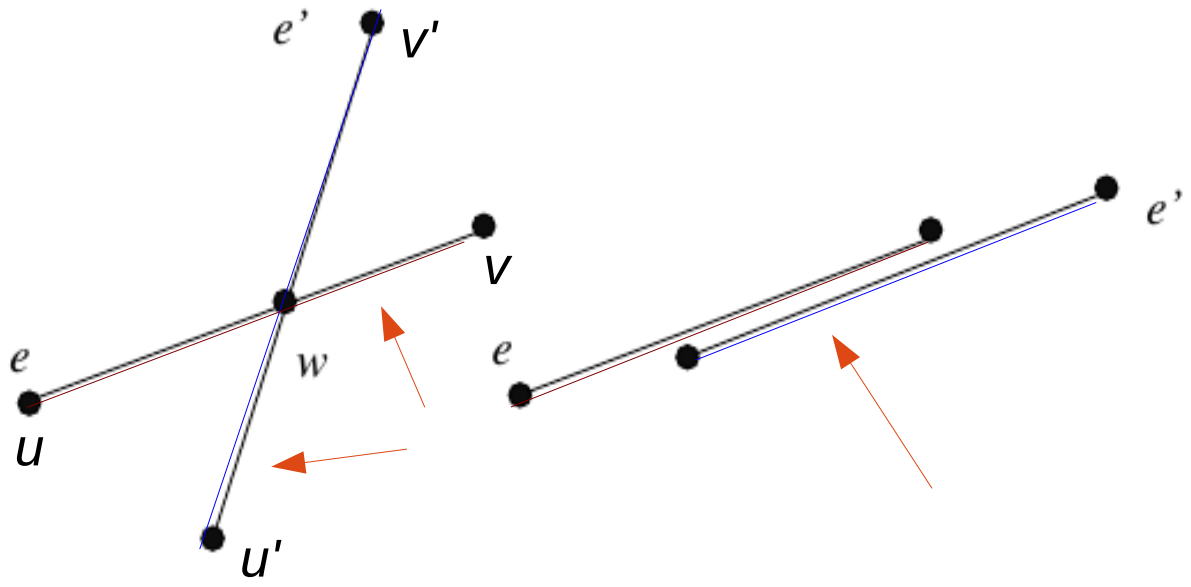


Propagação de Adjacências:

$$e_{ui} \cdot n_f < 0 \rightarrow e_{ui} \in A$$

$$\exists e_{ui} \cdot n_f > 0 \wedge e_{ui} \cdot n_f < 0 \rightarrow \exists 2\, arestas \in f$$

Interseção Aresta/Aresta



Arestas concorrentes

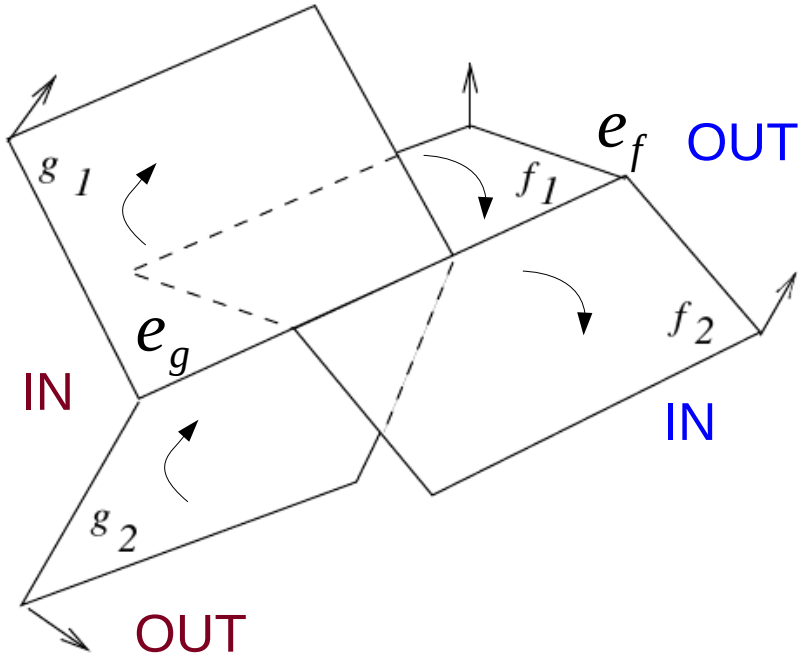
Arestas colineares

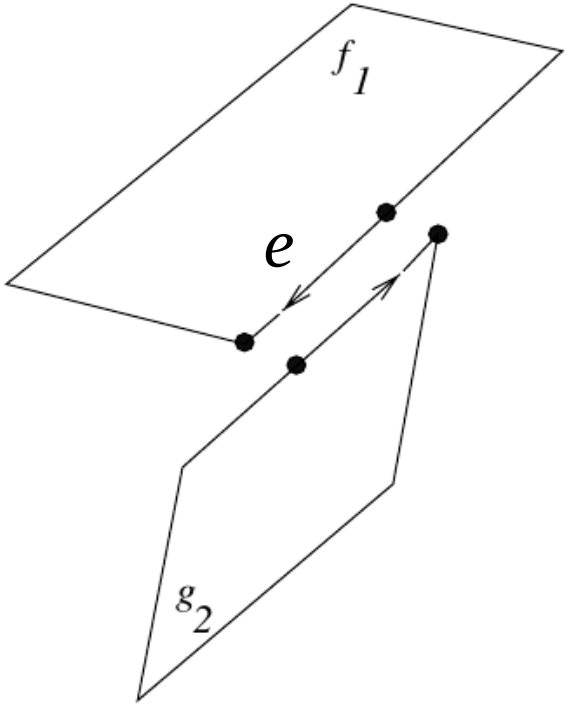
$(\overline{u'w}, \overline{wv'} \in g) \subset A\,?$

$(\overline{uw}, \overline{wv} \in f) \subset B\,?$

Determinar intervalos coincidentes

Propagação de Adjacências

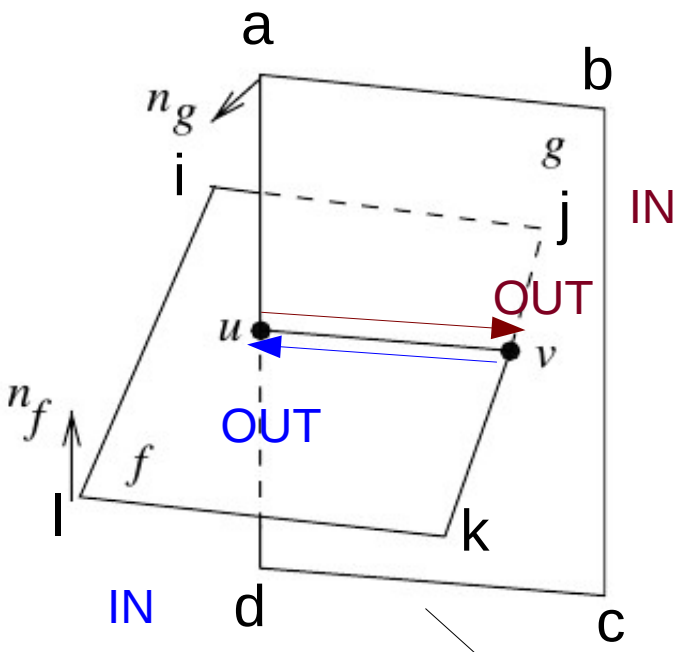


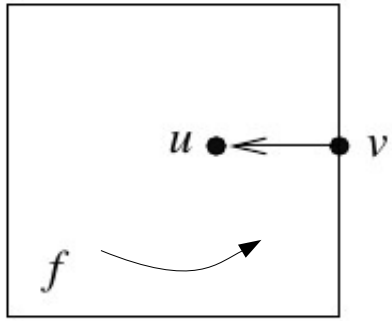


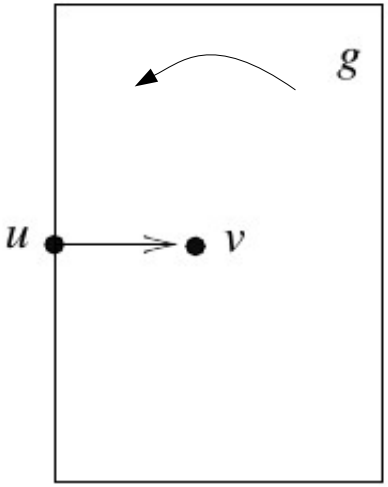
$$f_1 \subset Cunha(e_g) \rightarrow e \in \vartheta(f_1) \subset B$$

$$g_2 \subset Cunha(e_f) \rightarrow e \in \vartheta(g_2) \subset A$$

Interseção Faces Concorrentes







$\overline{jk} \cap g = v$

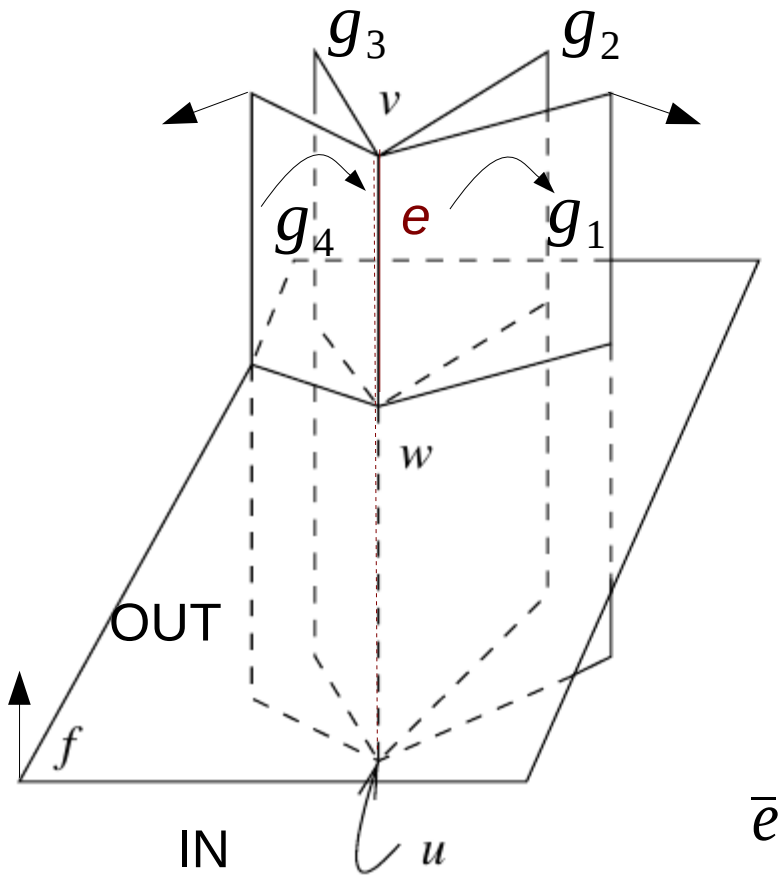
$\overline{ad} \cap f = u$

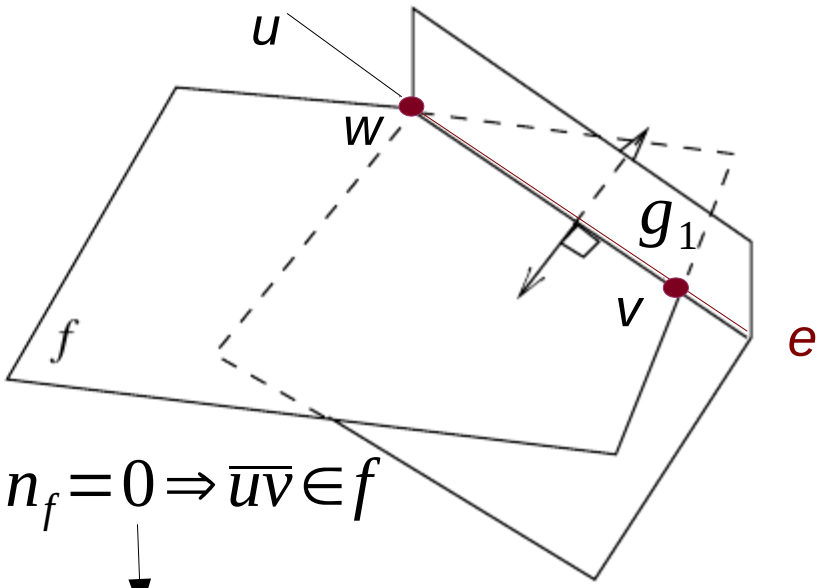
Pareie uv → $\overline{uv}$

Orientações de $\overline{uv}$

Avaliação da vizinhaça de u e v para agrupamento

Propagação para Adjacências





$$\overline{uv} \cdot n_f = 0 \Rightarrow \overline{uv} \in f$$

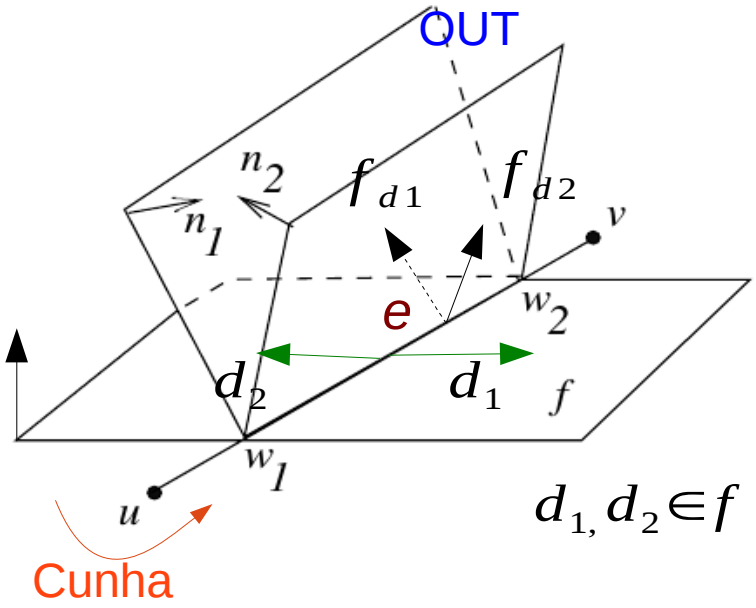
$$\bar{e} \cap \vartheta(f) = pontos \rightarrow segmentos \rightarrow f, adj(e)$$

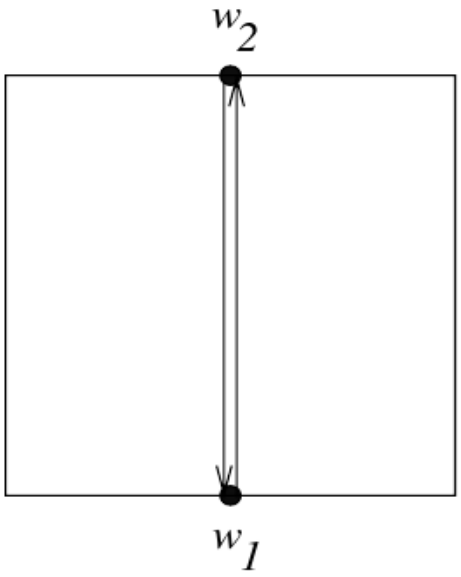
$$\overline{uv} \cdot n_f > 0 \Rightarrow \overline{uw} \in A$$

$$\overline{uv} \cdot n_f < 0 \Rightarrow \overline{wv} \in A$$

Validar as interseções (segmentos) das faces adjacentes a e → passam por w

Aresta sobre Face



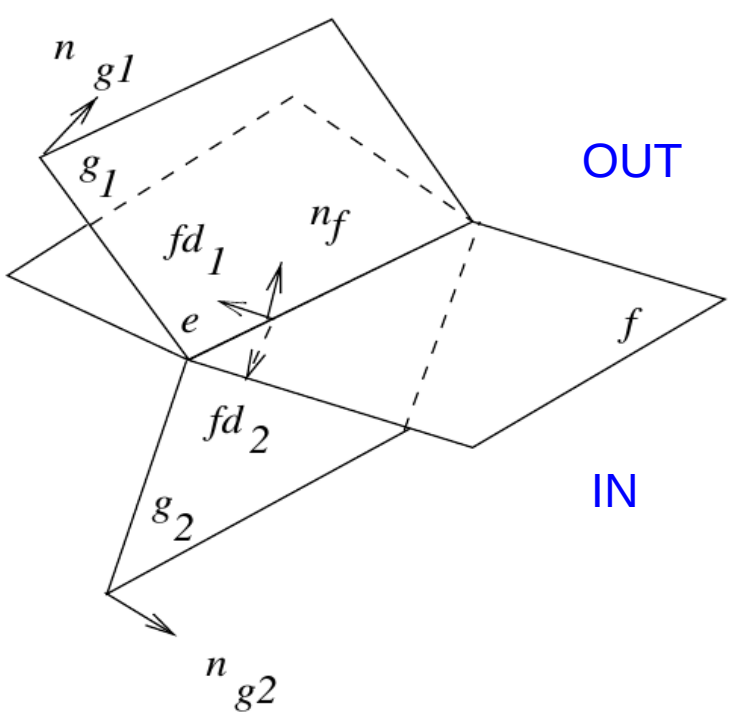


$$\bar{e} \cap \vartheta(f) = pontos \rightarrow segmentos \rightarrow f, adj(e)$$

$$sinal(f_{d1} \cdot d_1) = sinal(d_1 \cdot f_{d2}) \rightarrow f \subset B \rightarrow e \in f$$

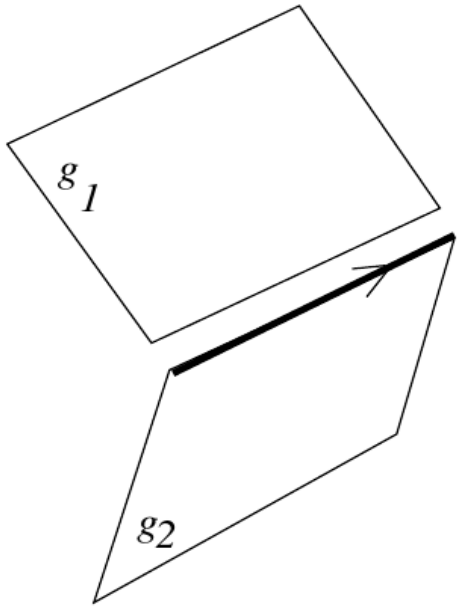
$$sinal(f_{d1} \cdot d_2) = sinal(d_2 \cdot f_{d2}) \rightarrow f \subset B \rightarrow e \in f$$

Aresta sobre Face



OUT

IN



$$\bar{e} \cap \vartheta(f) = pontos \rightarrow segmentos \rightarrow f, adj(e)$$

$$f_{d1} \cdot n_f > 0 \rightarrow g_1 \not\subset A$$

$$f_{d2} \cdot n_f < 0 \rightarrow g_2 \subset A \rightarrow e \in \vartheta(g_2)$$

Subdivisão de Faces

1. Initialize a list L of all points in f.

2. If L is empty, stop. Otherwise, initialize the stack S to contain a point u in L.

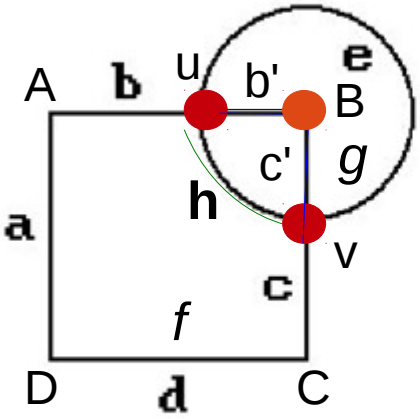
3. If S is empty, return to step 2. Otherwise, pop u from S and delete it from L. Mark u as explored.

4. Let E_1 be the set of all segments incident to u contained in f. If u is not a point, then let E_2 be all edges of f not containing a point; otherwise, E_2 is empty.

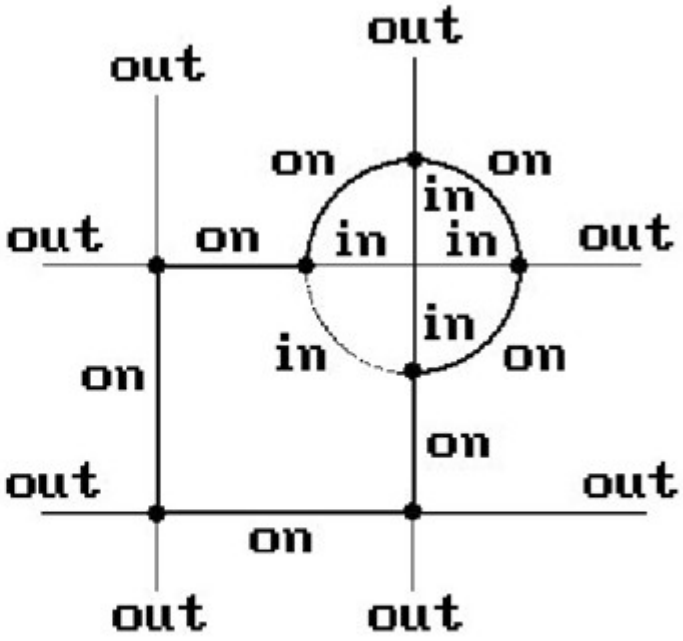
5. Order the edges and segments in $E_1 \cup E_2$ cyclically about u in the plane of f, and construct area-enclosing pairs.

6. For each (u, w) or (w, u) in $E_1 \cup E_2$, stack w if it is unexplored. Then return to step 3.

Grafo de Incidências



(a)



(b)

u, v : pontos de *f*
B, u e v: pontos de *g*.
A, B, C e D: vértices de *f*

→

f: E1={b,b',c,c'},
E2={a,d}
g: E1={b',c',e,h},
E2={}

→

Adj(u)={b,b',h}
Adj(v)={c',h,e}
Adj(B)={b',c'}

→

f: {a,b,h,c,d}
{b', c', h}
g: {e,c',b}
{b', c', h}

Agregação de Adjacências

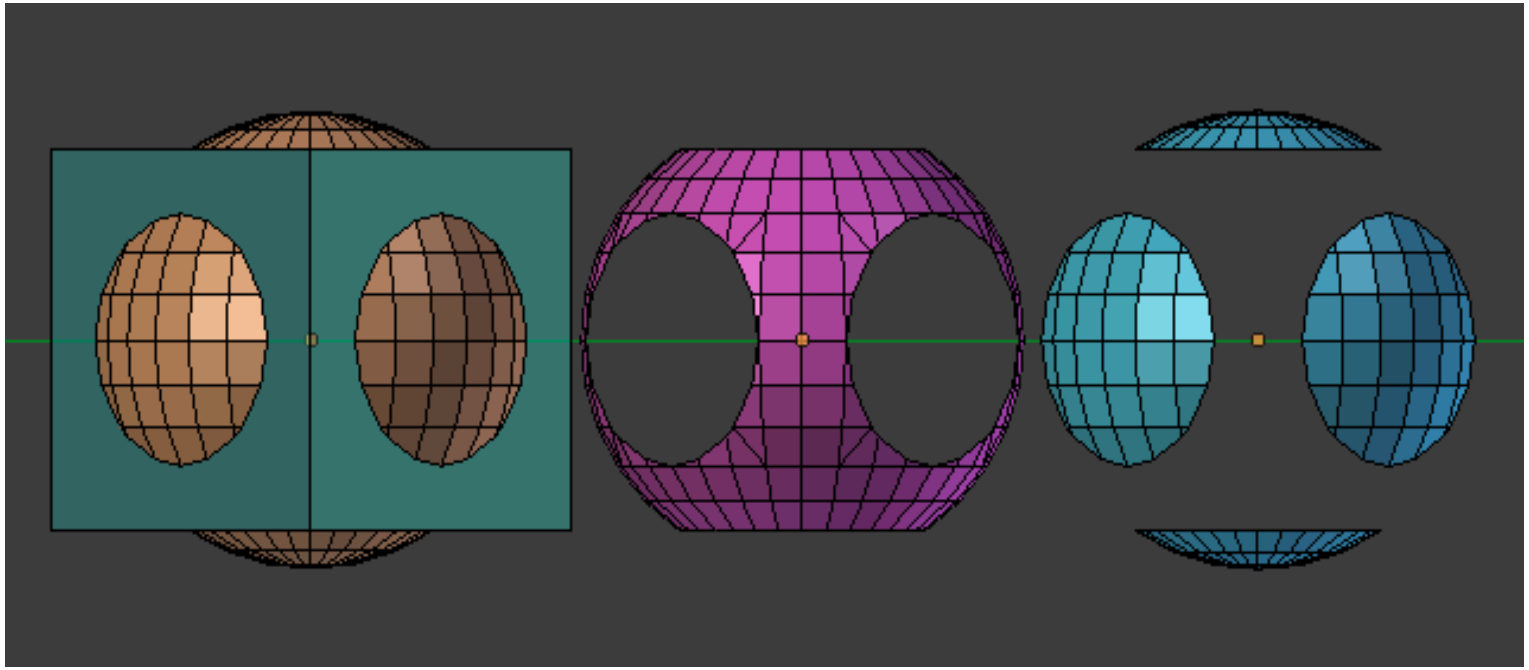
1. Let F_1 be the set of all faces of $A \cap^* B$ constructed by the subdivision given previously, and mark them as unprocessed. Set F_2 to the empty set.

2. If all faces in $F_1 \cup F_2$ have been processed, then stop. We have found all faces of $A \cap^* B$.

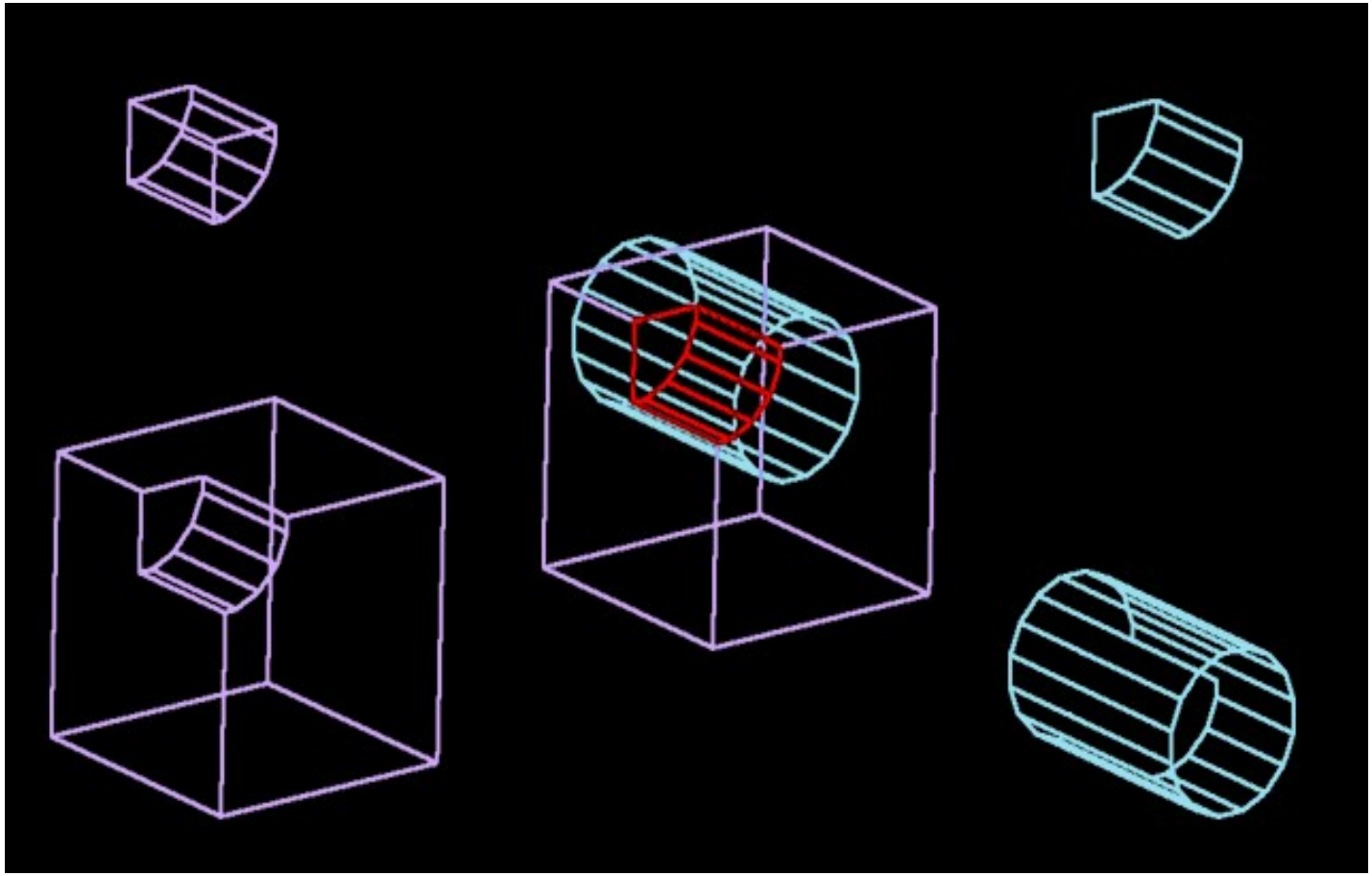
3. For all unprocessed faces f in $F_1 \cup F_2$, mark f as processed. For each edge (u, v) of f where u is not a point, add to F_2 all faces incident to u in A or in B that have not been subdivided, and mark them as unprocessed.

F1: faces na borda das interseções
F2: faces interiores

Agrupamento de pontos e segmentos



Identificação de Faces Adjacentes



Pseudo-códigos de Algoritmos de Interseção

http://geomalgorithms.com/a09-_intersect-3.html#Pseudo-Code:%20S-H