IA841 – Modelagem de Sólidos

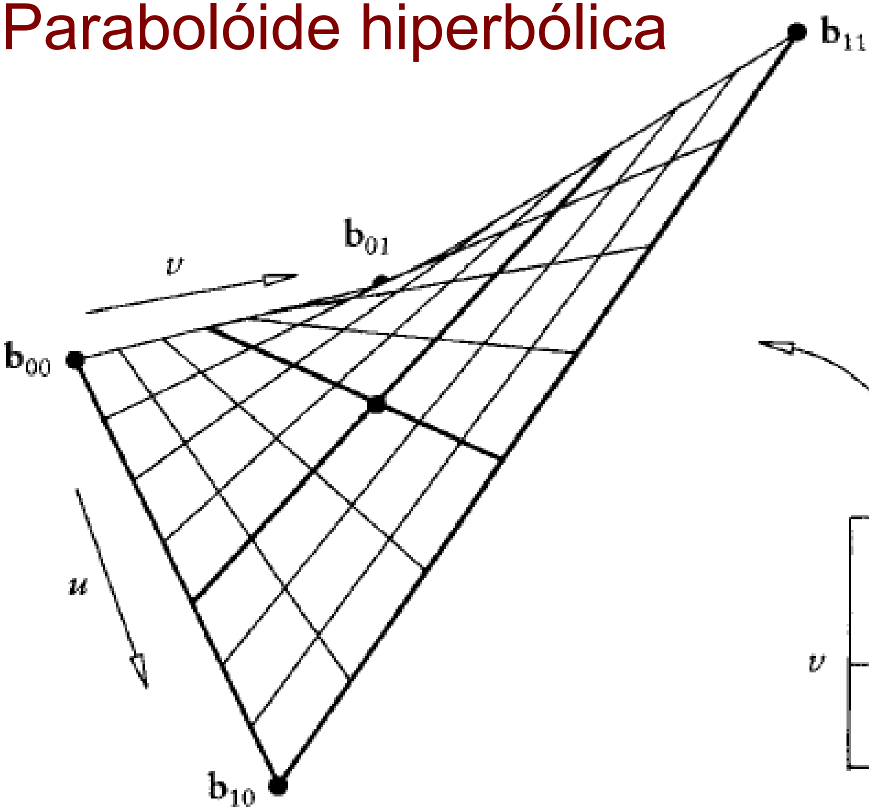
Superfícies

Farin: Capítulos 14, 16, 17, 18

Produto Tensorial de Duas Curvas

- Interpolação Bilinear

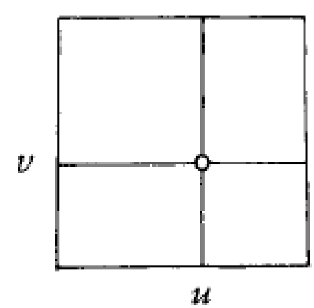
Parabolóide hiperbólica



Contradomínio

Função Interpoladora

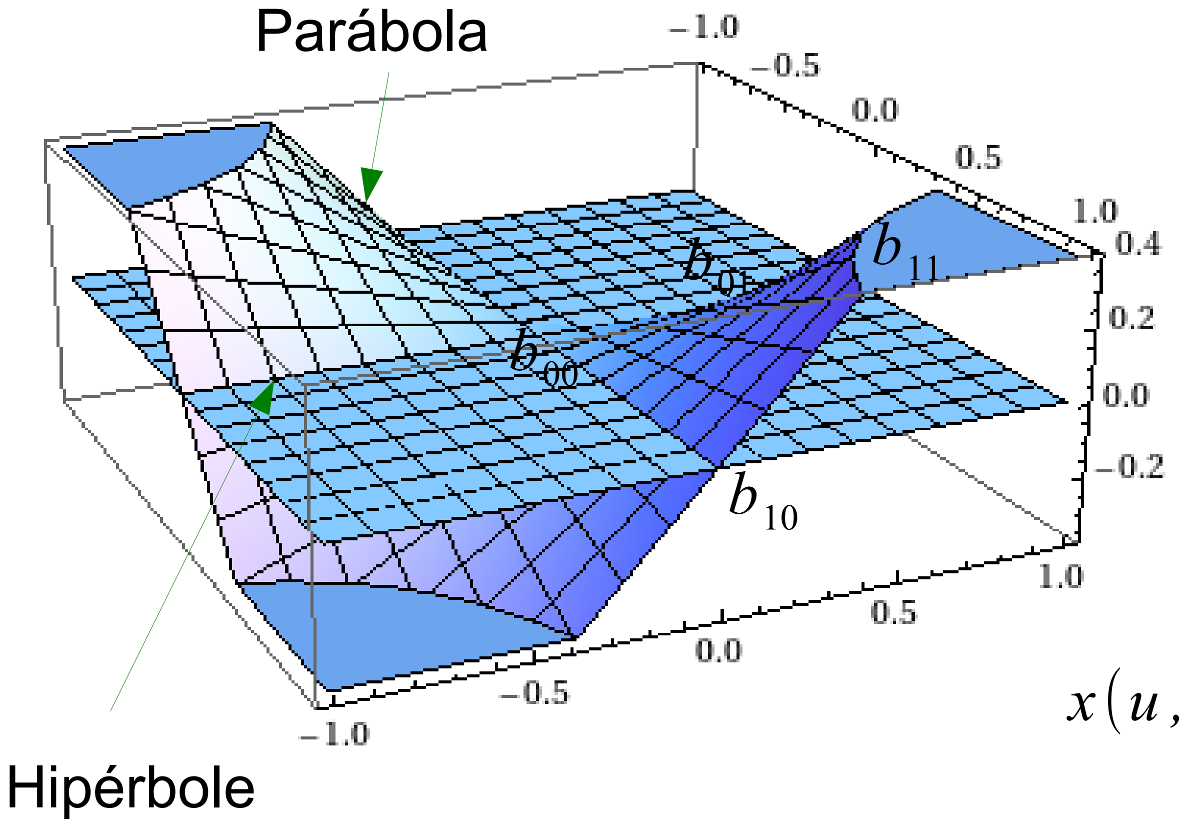
$$x(u,v)=\begin{pmatrix}1-u & u\end{pmatrix}\begin{pmatrix}b_{00} & b_{01}\\ b_{10} & b_{11}\end{pmatrix}\begin{pmatrix}1-v\\ v\end{pmatrix}$$



Domínio

Interpolação Bilinear

Representação Implícita: z=xy



$$b_{00}^{01}=(1-v)b_{00}+vb_{01}$$

$$b_{10}^{11}=(1-v)b_{10}+vb_{11}$$

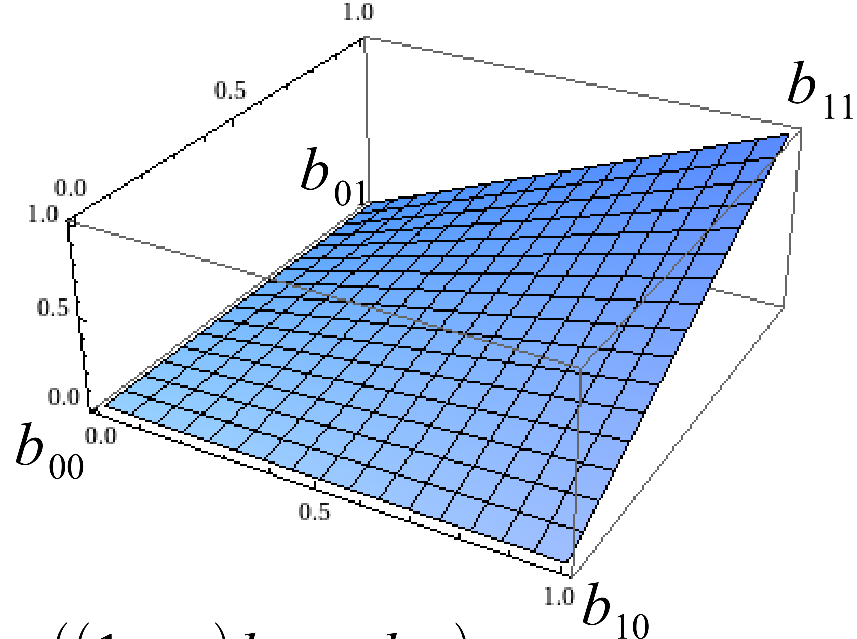
$$x(u,v)=b_{00}^{11}=(1-u)b_{00}^{01}+ub_{10}^{11}$$

Simetria

$$b_{00}^{01}=(1-v)b_{00}+vb_{01}$$

$$b_{10}^{11}=(1-v)b_{10}+vb_{11}$$

$$x(u,v)=b_{00}^{11}=(1-u)b_{00}^{01}+ub_{10}^{11}$$



$$x(u,v)=b_{00}^{11}=(1-u)((1-v)b_{00}+v\,b_{01})+u((1-v)b_{10}+vb_{11})$$

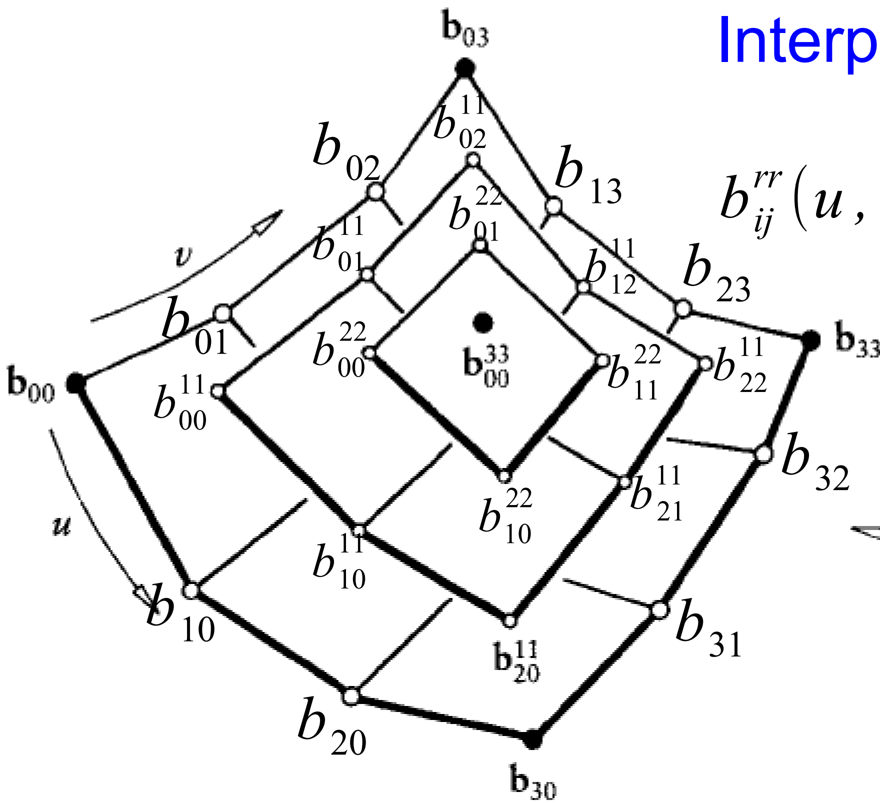
$$x(u,v)=b_{00}^{11}=(1-v)(1-u)b_{00}+v(1-u)b_{01}+(1-v)u\,b_{10}+v\,u\,b_{11}$$

$$x(u,v)=b_{00}^{11}=(1-v)((1-u)b_{00}+u\,b_{10})+v((1-u)b_{01}+u\,b_{11})$$

$$x(u,v)=b_{00}^{11}=(1-v)b_{00}^{10}+vb_{01}^{11}$$

Algoritmo de DeCasteljau

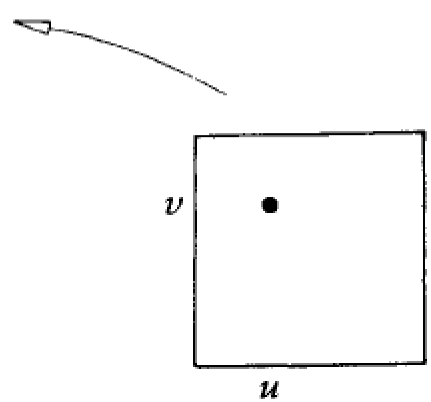
- Superfícies de grau nxn → (n+1)x(n+1) pontos



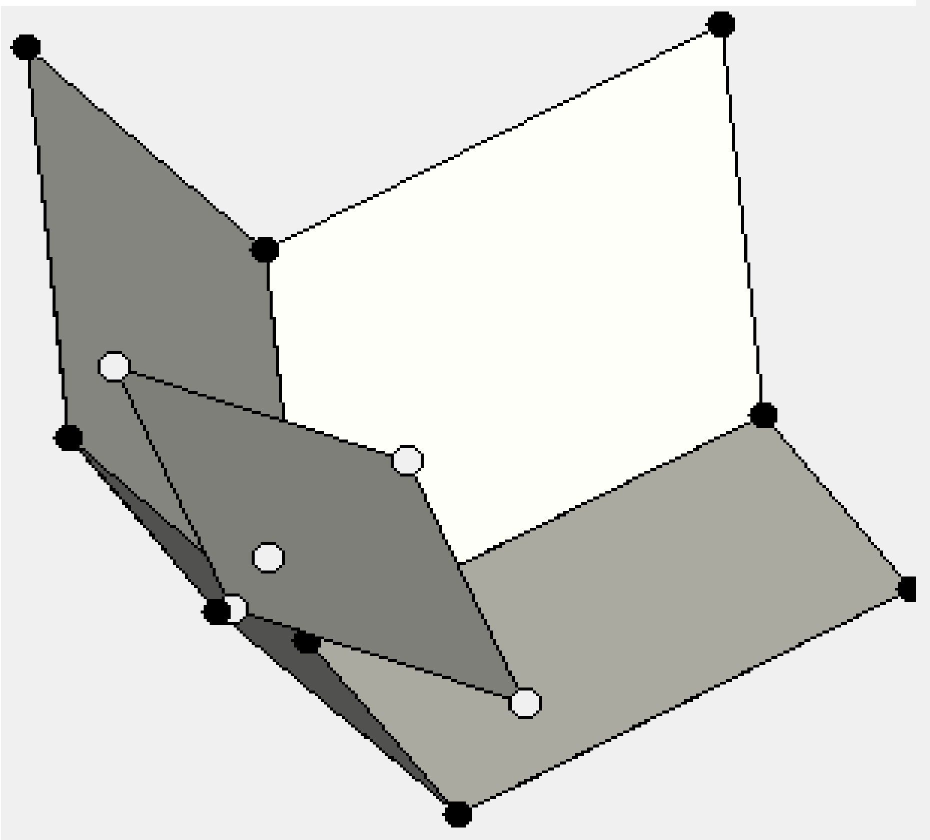
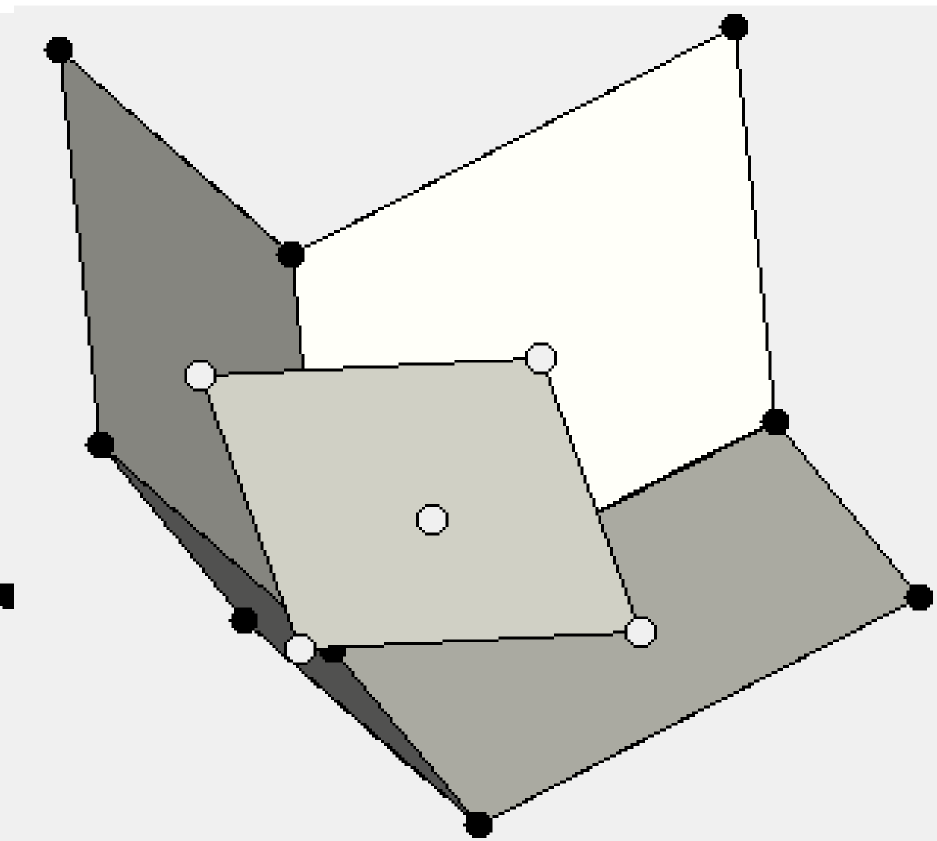
Interpolações bilineares por partes

$$b_{ij}^{rr}(u,v) = \begin{pmatrix} 1-u & u \end{pmatrix} \begin{pmatrix} b_{ij}^{r-1\,r-1} & b_{ij+1}^{r-1\,r-1} \\ b_{i+1\,j}^{r-1\,r-1} & b_{i+1\,j+1}^{r-1\,r-1} \end{pmatrix} \begin{pmatrix} 1-v \\ v \end{pmatrix}$$

$b_{00}^{nn}(u,v)$: ponto sobre a superfície

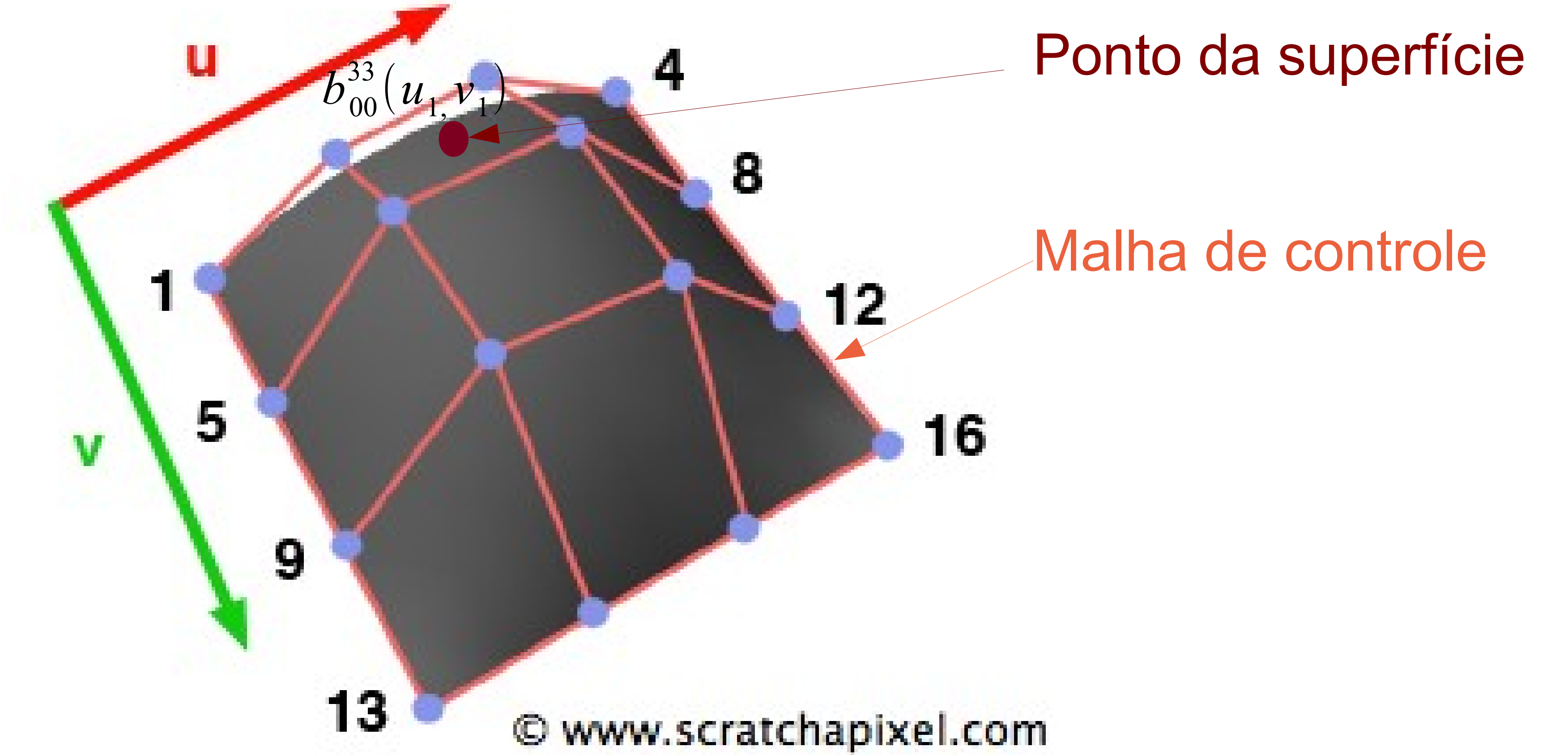


Exemplos

http://cagd-applets.webarchiv.kit.edu/mocca/html/noplugin/TPBezierSurface/AppDeCasteljau/index
.html

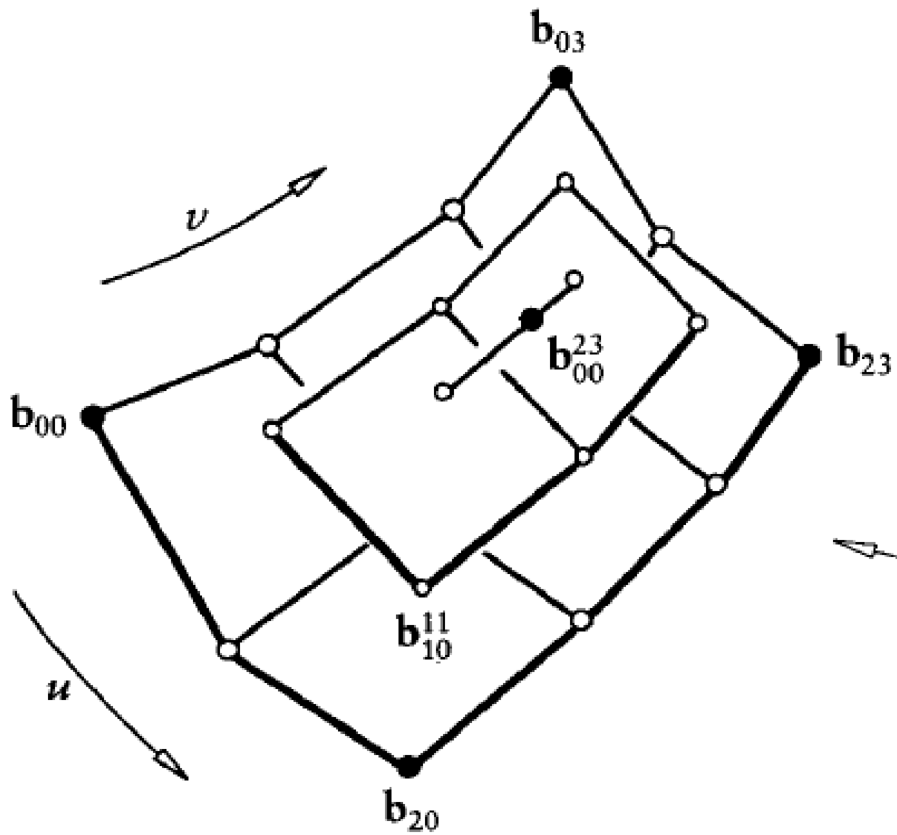
Superfície de Bézier



http://www.scratchapixel.com/old/lessons/3d-basic-lessons/lesson-11-rendering-the-teapot-bezier-surfaces/b-zier-s rface/

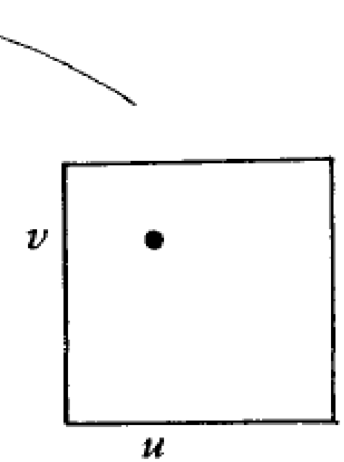
Malha Retangular

- Superfícies de grau nxm → (n+1)x(m+1) pontos



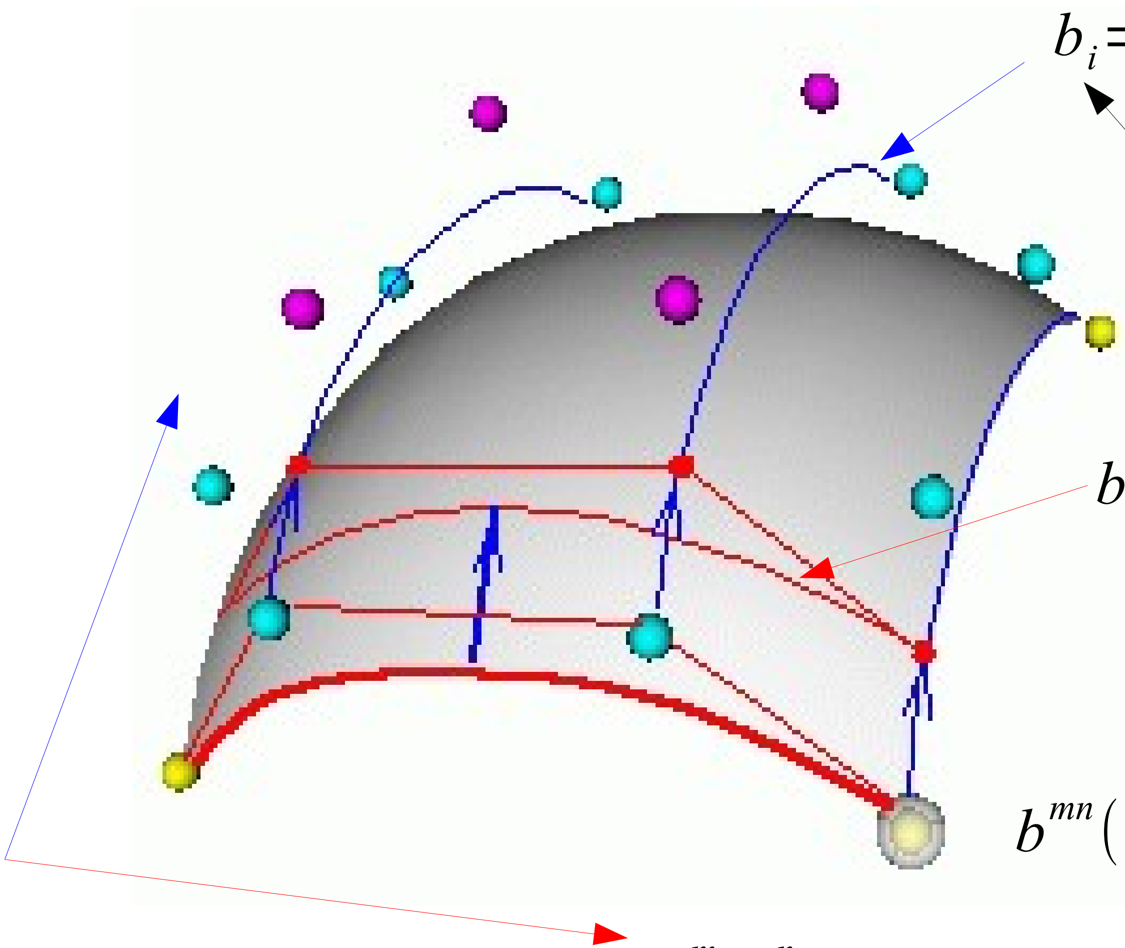
k=min(m,n); l=max(m,n)

(1) interpolações bilineares até b_{ij}^{kk}
(2) (l-m) interpolações lineares



Produto Tensorial

$$b_i = b_i^n(v) = \sum_{j=0}^{n} b_{ij} B_j^n(u)$$

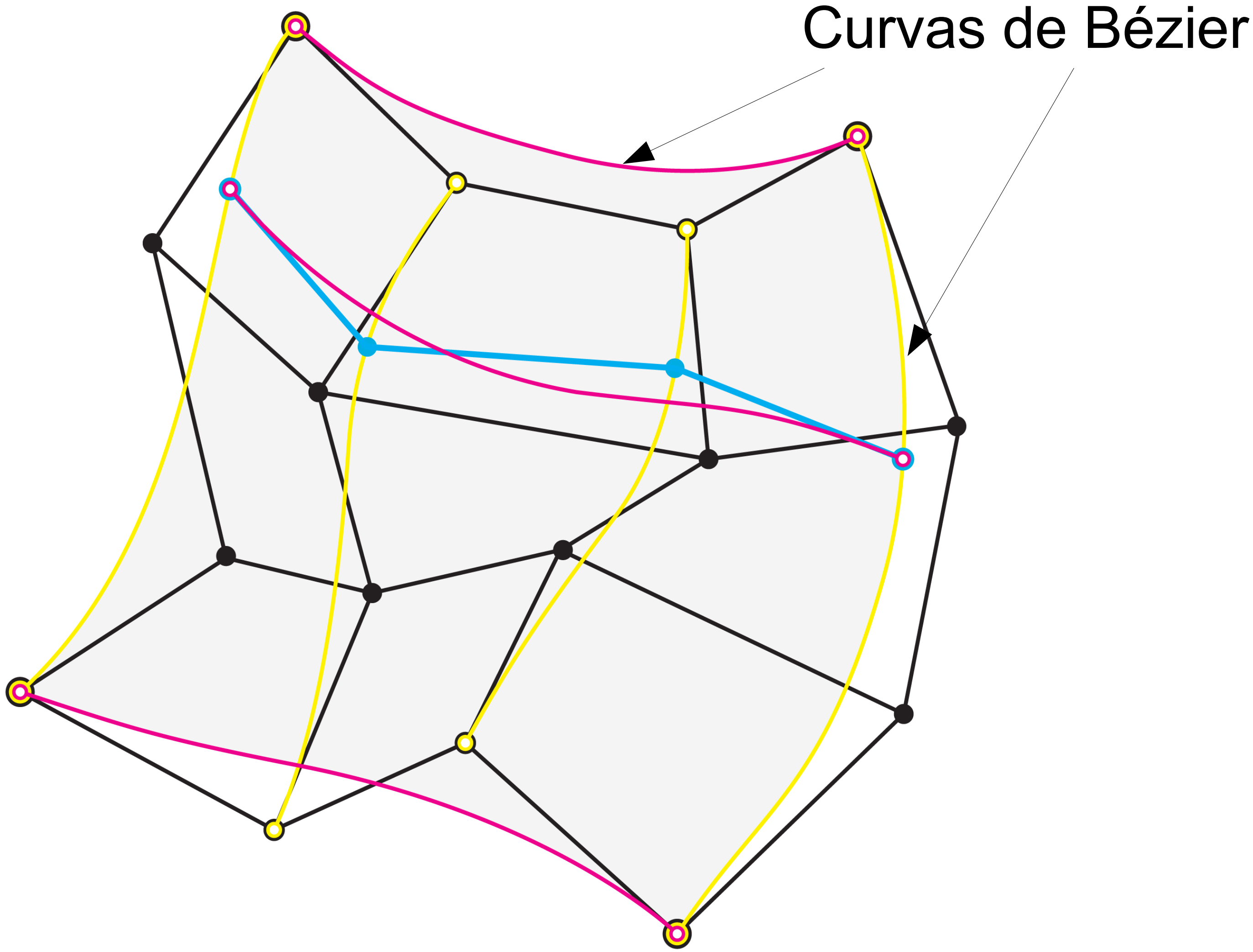


$$b^m(u) = \sum_{i=0}^{m} b_i B_i^m(u)$$

$$b^{mn}(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} b_{ij} B_j^n(v) B_i^m(u)$$

$$\sum_{i=0}^{m} \sum_{j=0}^{n} B_j^n(v) B_i^m(u) = \left(\sum_{i=0}^{m} B_i^m(u)\right)\left(\sum_{j=0}^{n} B_j^n(v)\right) = 1$$

Borda do Produto Tensorial



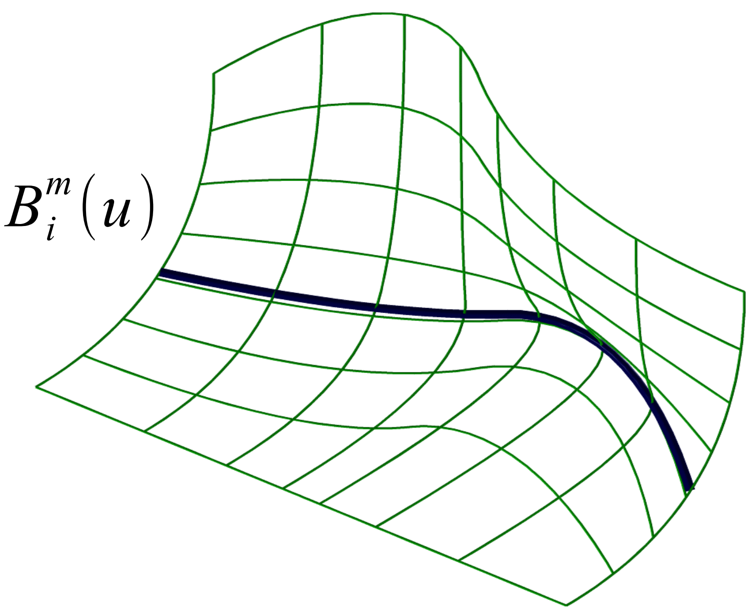
Curvas Isoparamétricas

- u constante $\rightarrow \hat{u}$

$$b^{mn}(\hat{u},v)=\sum_{j=0}^{n}(\sum_{i=0}^{m} b_{ij}B_i^m(\hat{u}))B_j^n(v)=\sum_{j=0}^{n} b_{i0}^{0\mathrm{n}}B_j^n(v)$$

- v constante $\rightarrow \hat{v}$

$$b^{mn}(u,\hat{v})=\sum_{i=0}^{m}(\sum_{j=0}^{n} b_{ij}B_j^n(\hat{v}))B_i^m(u)=\sum_{i=0}^{m} b_{0\mathrm{j}}^{m0}B_i^m(u)$$



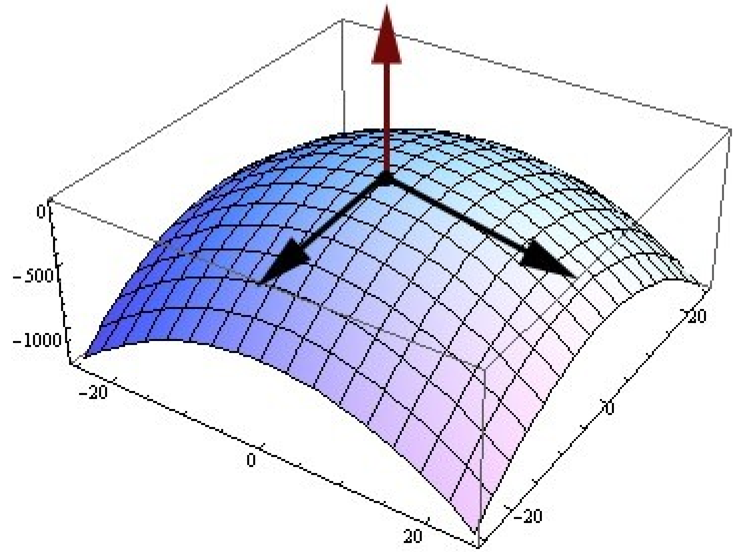
Propriedades

- Invariantes sob transformações afins.
- Convexidade.
- Interpolam os quatro vértices da malha de controle
- Satisfazem a propriedade de *variation diminishing*.

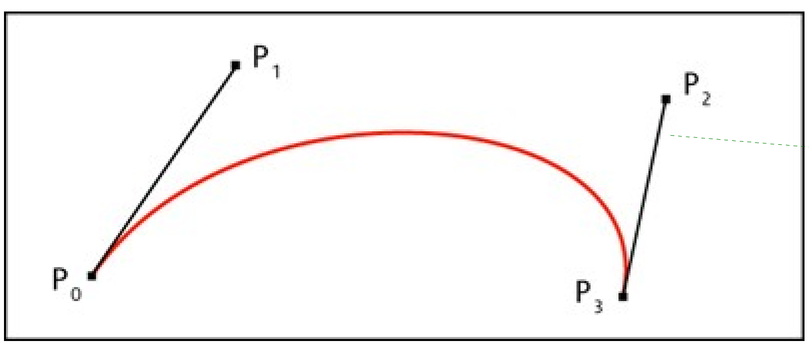
Vetores Normais

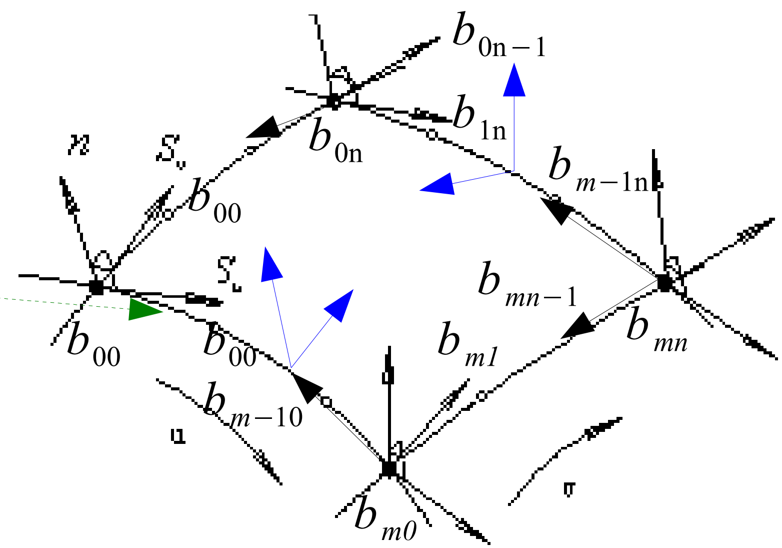
- Pontos interiores

$$\vec{n}(u,v)=\frac{\dfrac{\partial b^{mn}(u,v)}{\partial u}\times\dfrac{\partial b^{mn}(u,v)}{\partial v}}{\left\|\dfrac{\partial b^{mn}(u,v)}{\partial u}\times\dfrac{\partial b^{mn}(u,v)}{\partial v}\right\|}$$

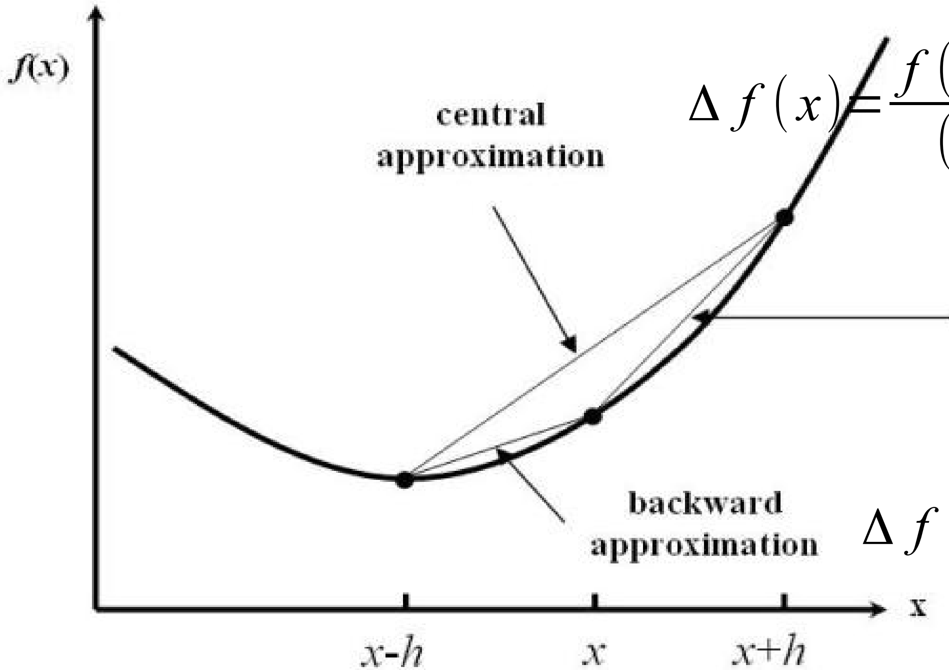


- Borda e vértices





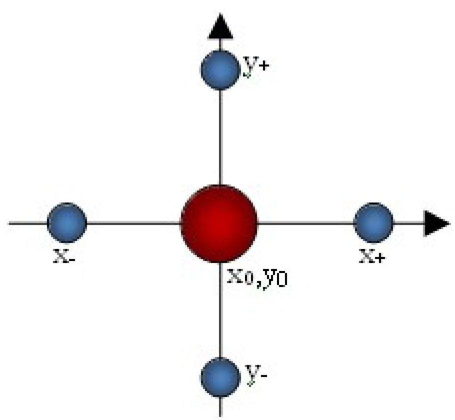
Diferenças Finitas

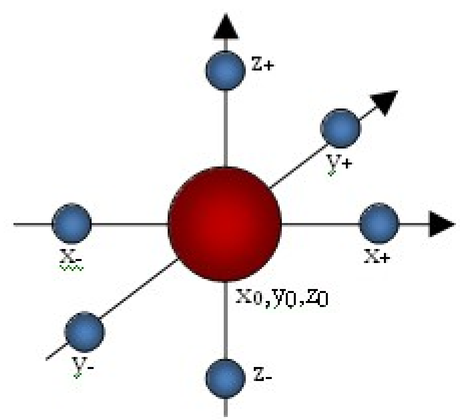


$$\Delta f(x) = \frac{f(x+h) - f(x-h)}{(x+h) - (x-h)}$$

$$\Delta f(x) = \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$\Delta f(x) = \frac{f(x) - f(x-h)}{x - (x-h)}$$





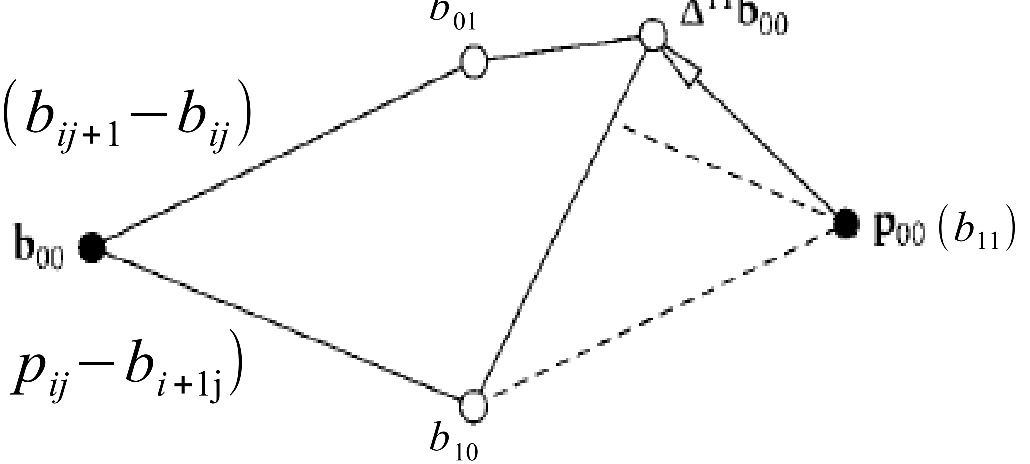
Torsão (*Twists*)

- Mede o quanto a malha de controle de Bézier se desloca do paralelogramo formado por $b_{ij}, b_{i+1j}, b_{ij+1}$

$$\Delta^{10} b^{mn}(i,j) = \frac{b_{i+1j} - b_{ij}}{\Delta i} = b_{i+1j} - b_{ij}$$

$$\Delta^{01} b^{mn}(i,j) = \frac{b_{ij+1} - b_{ij}}{\Delta j} = b_{ij+1} - b_{ij} = p_{ij} - b_{i+1j}$$

$$\Delta^{11} b^{mn}(i,j) = (b_{i+1j+1} - b_{i+1j}) - (b_{ij+1} - b_{ij})$$



$$\Delta^{11} b^{mn}(i,j) = (b_{i+1j+1} - b_{i+1j}) - (p_{ij} - b_{i+1j})$$

$$\Delta^{11} b^{mn}(i,j) = b_{i+1j+1} - p_{ij}$$

Notação Matricial

$$b^{mn}(u,v)=\begin{pmatrix} B_0^m(u) & \cdots & B_m^m(u) \end{pmatrix}\begin{pmatrix} b_{00} & \cdots & b_{0\mathrm{n}} \\ \vdots & \ddots & \vdots \\ b_{m0} & \cdots & b_{mn} \end{pmatrix}\begin{pmatrix} B_0^m(v) \\ \vdots \\ B_m^m(v) \end{pmatrix}$$

Superfícies Bicúbicas

$$\begin{pmatrix} B_0^3(v) \\ B_1^3(v) \\ B_2^3(v) \\ B_3^3(v) \end{pmatrix}=\begin{pmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ v \\ v^2 \\ v^3 \end{pmatrix}$$

$$\begin{pmatrix} B_0^3(u) & B_1^3(u) & B_2^3(u) & B_3^3(u) \end{pmatrix}=\begin{pmatrix} 1 & u & u^2 & u^3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{pmatrix}$$

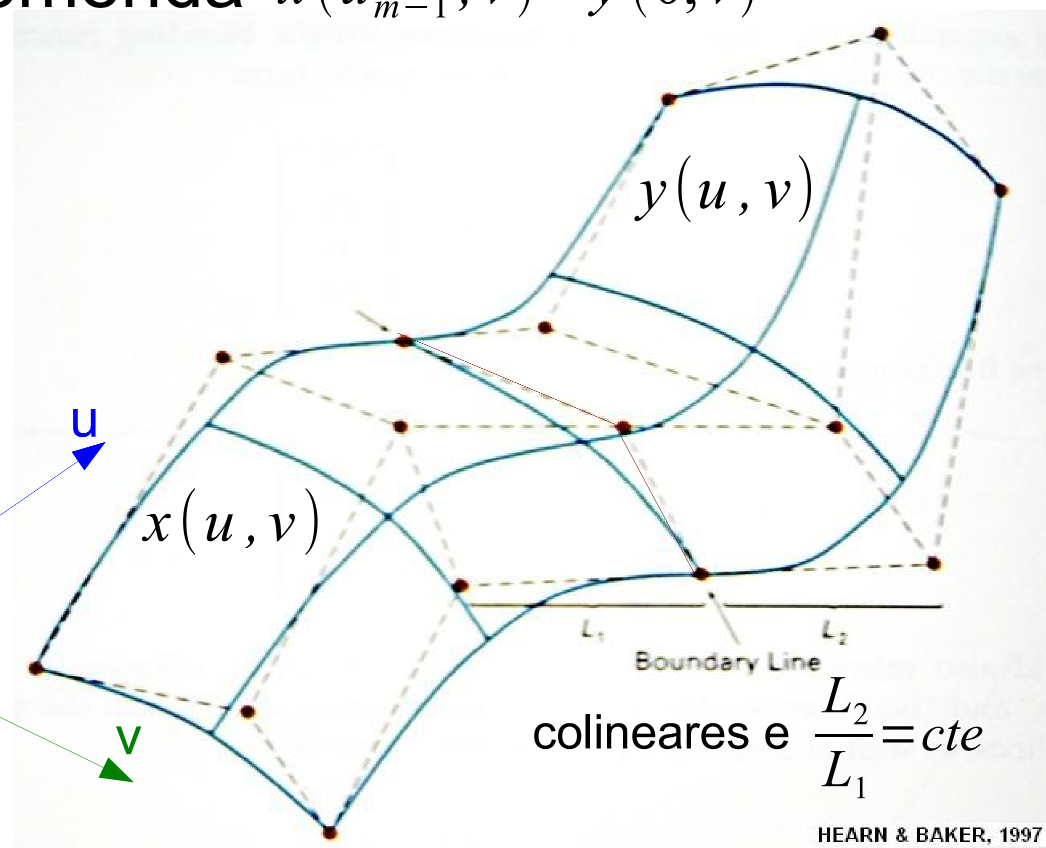
Continuidade C^n

- Diferenciabilidade ou derivabilidade até ordem n
- Continuidade C^1 na emenda $x(u_{m-1}, v) = y(0, v)$

$$\frac{\partial x(u_{m-1}, v)}{\partial u} = \frac{\partial y(0, v)}{\partial u}$$

$$\Downarrow$$

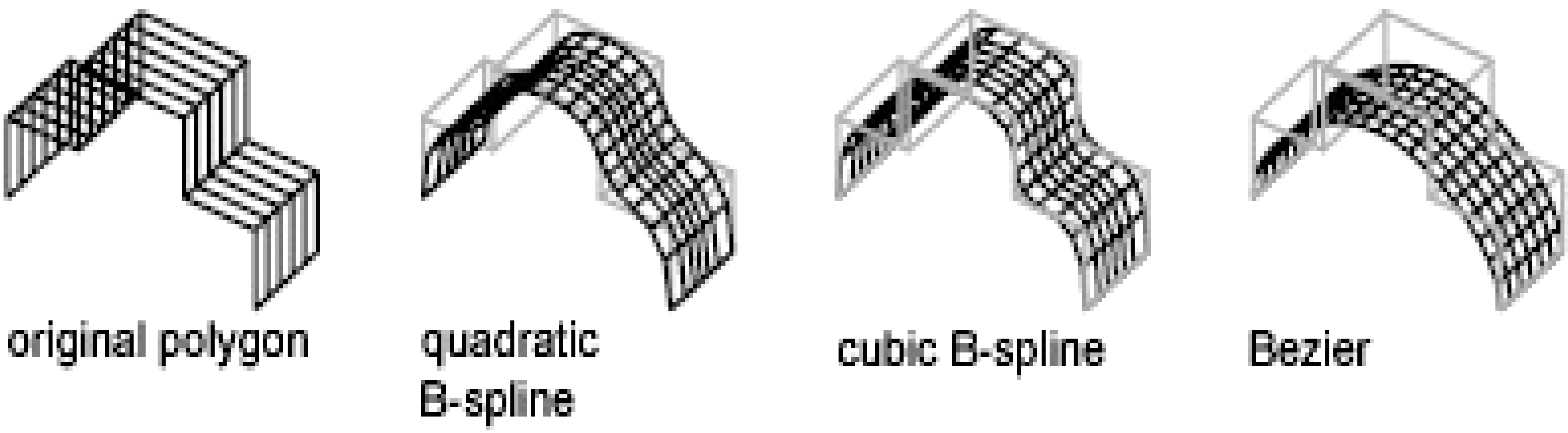
Continuidade C^1
em todas curvas
isoparamétricas v



colineares e $\frac{L_2}{L_1} = cte$

HEARN & BAKER, 1997

Superfícies de *B-Spline*



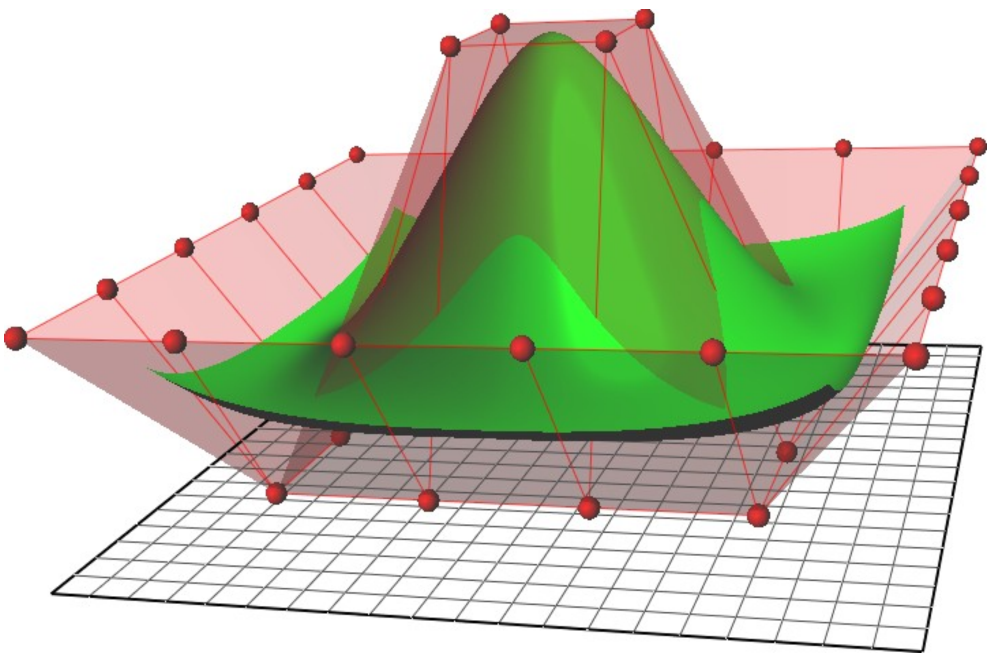
$$x(u,v)=\sum_{i=0}^{m}\sum_{j=0}^{n} d_{ij} N_j^n(v) N_i^m(u)=\sum_{i=0}^{m}\left(\sum_{j=0}^{n} d_{ij} N_j^n(v)\right) N_i^m(u)$$

É assegurada a continuidade C^{n-1} ao longo das curvas isoparamétricas sem nós múltiplos.

Superfícies Racionais

- Não são formadas pelos produtos tensoriais. São projeções de produtos tensoriais

$$x(u,v)=\frac{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\, b_{ij}\, B_j^n(v)\, B_i^m(u)}{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\, B_j^n(v)\, B_i^m(u)}$$

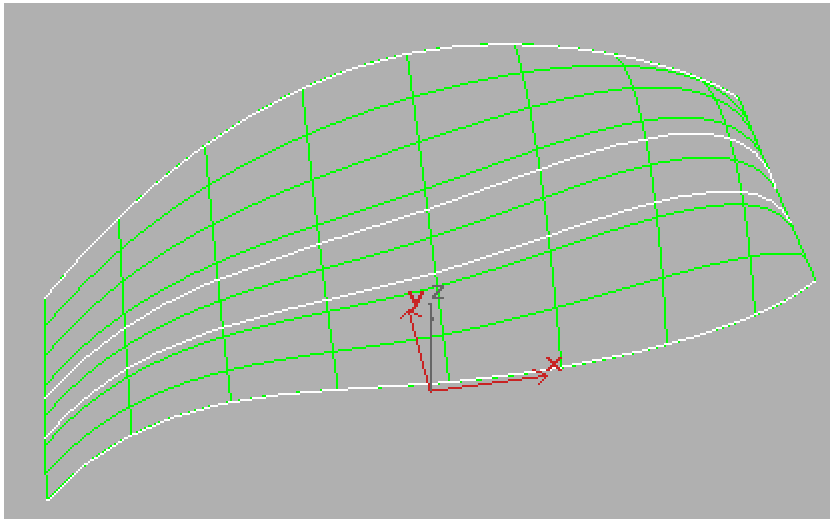


NURBS

$$x(u,v)=\frac{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\, d_{ij}\, N_j^n(v)\, N_i^m(u)}{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\, N_j^n(v)\, N_i^m(u)}$$

http://de.wikipedia.org/wiki/Non-Uniform_Rational_B-Spline

Curvas Isoparamétricas em NURBS

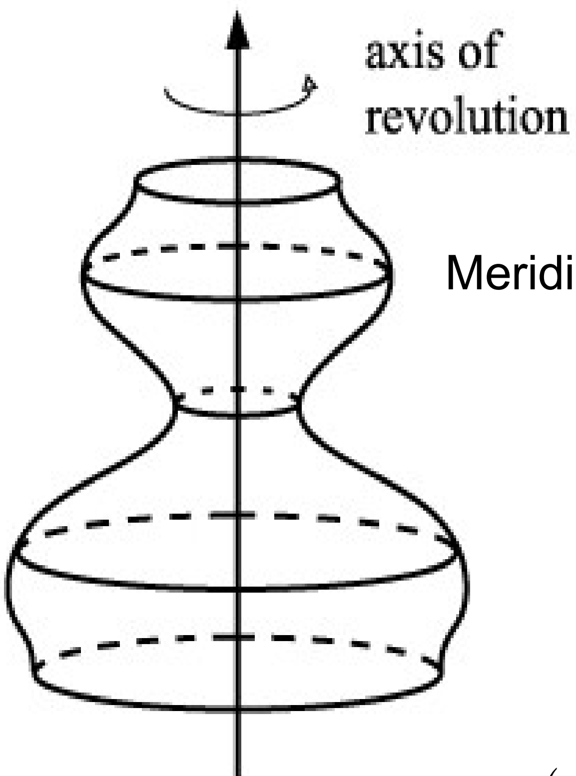


$$x(u,v)=\frac{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\, d_{ij}\, N_j^n(v)\, N_i^m(u)}{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\, N_j^n(v)\, N_i^m(u)}$$

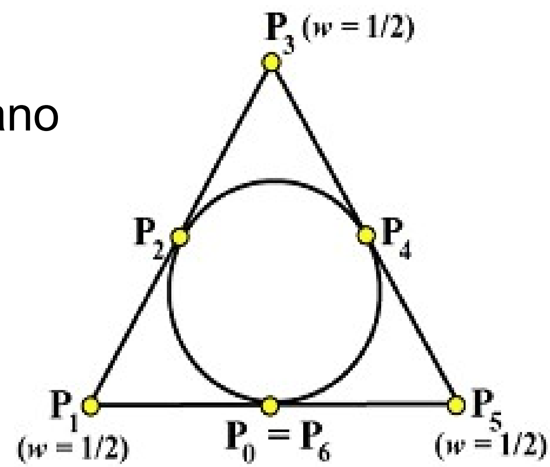
$$x(u,v)=\frac{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\, d_{ij}\, N_j^n(v)}{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}\, N_j^n(v)\, N_i^m(u)}\, N_i^m(u)$$

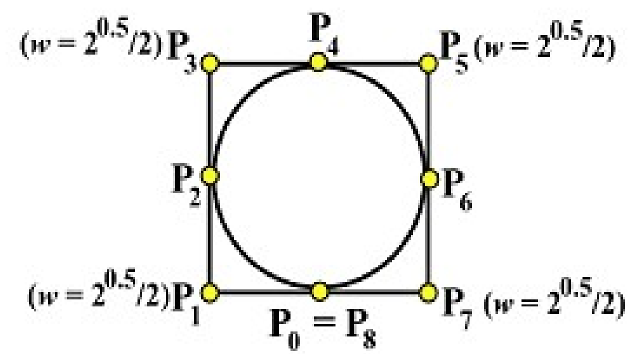
Superfícies de Revolução

$$x(u,v)=\begin{pmatrix} r(v)\cos u \\ r(v)\,sen\,u \\ z(v) \end{pmatrix}$$



Meridiano

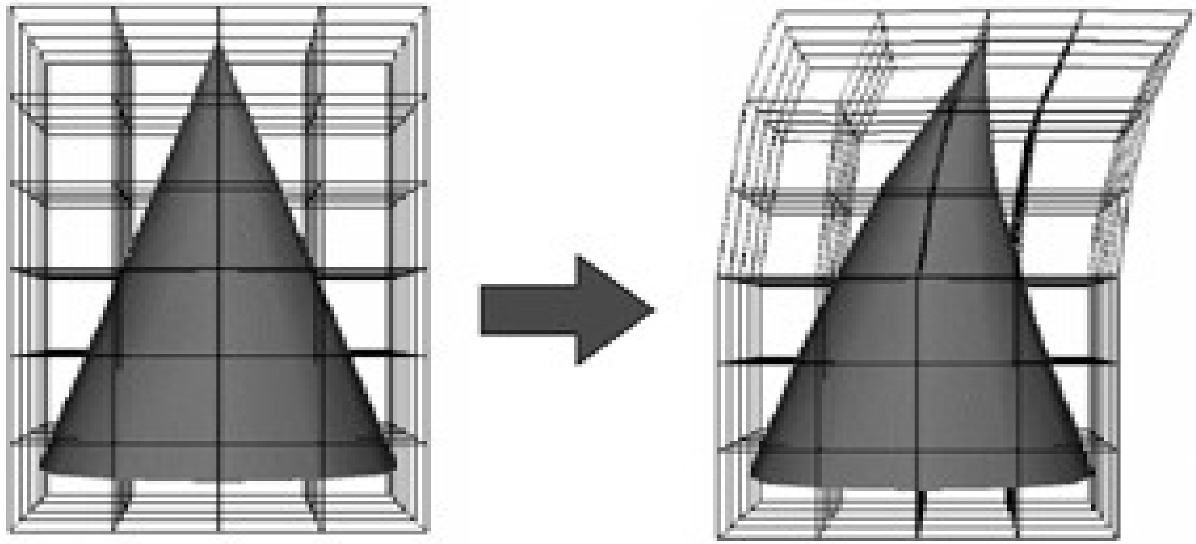




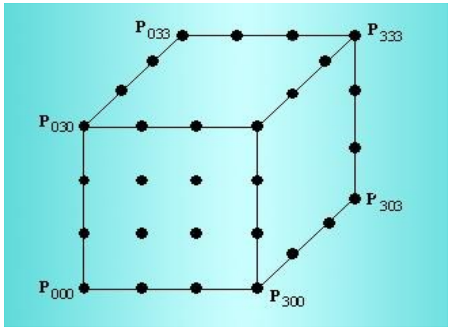
$$x(u,v)=\frac{P_0(v)\,B_0^2(u)+0.5\,P_1(v)\,B_1^2(u)+P_2(v)\,B_2^2(u)}{B_0^2(u)+0.5\,B_1^2(u)+B_2^2(u)}$$

http://www.cs.mtu.edu/~shene/COURSES/cs3621/NOTES/spline/NURBS/RB-circles.html

Volumes Bézier



$$(x, y, z) \Rightarrow (u, v, w)$$



$$(\hat{u}, \hat{v}, \hat{w}) \Rightarrow (\hat{x}, \hat{y}, \hat{z})$$

$$b^{mnl}(u, v, w) = \sum_{i=0}^{m} \sum_{j=0}^{n} \sum_{k=0}^{l} b_{ijk} B_i^m(u) B_j^n(v) B_k^l(w)$$

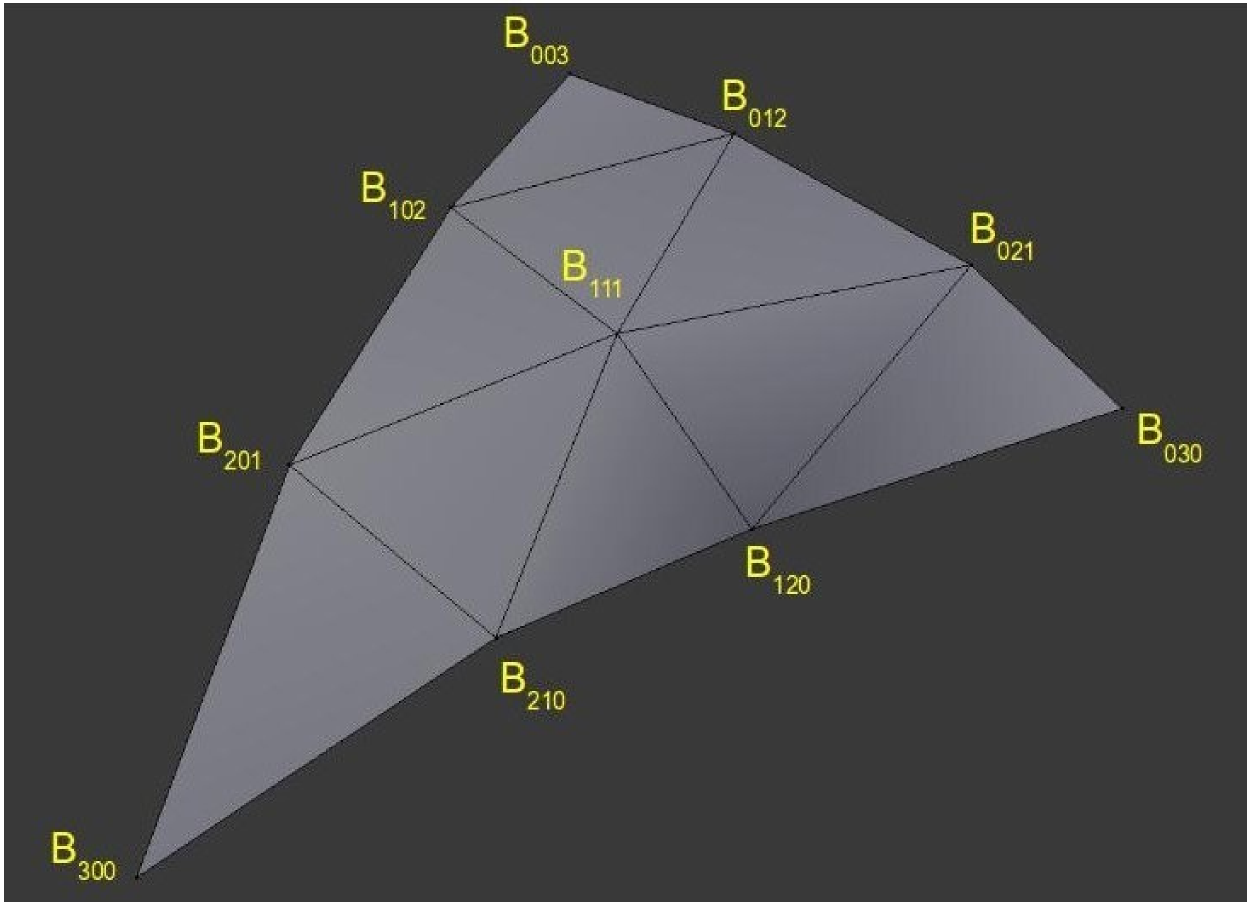
http://www.gamasutra.com/view/feature/3372/realtime_softobject_animation_.php?print=1

Triângulos de Bézier

Grau: n
Números de vértices:

Soma dos índices
dos vértices:

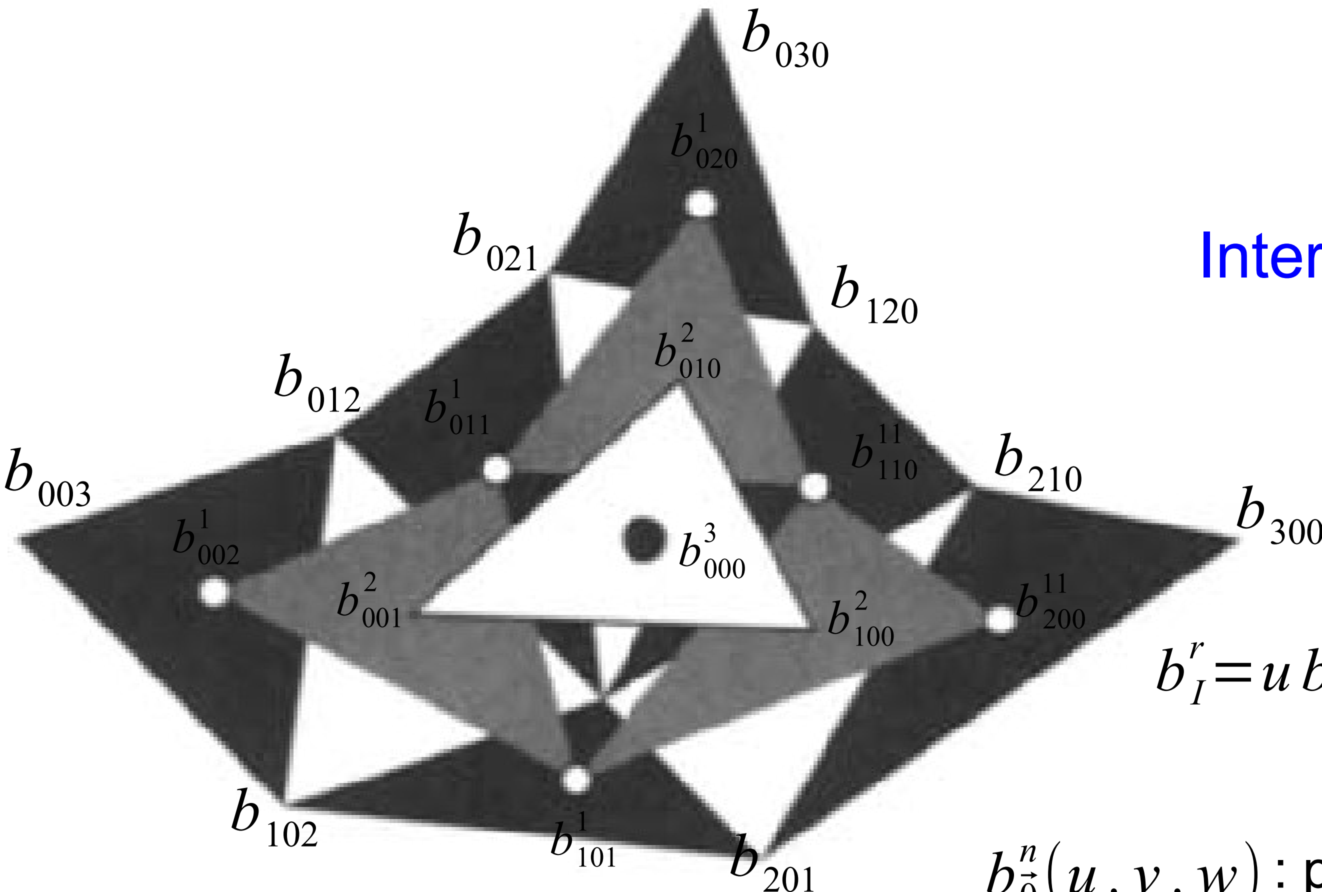
$$i + j + k = n$$



http://ogldev.atspace.co.uk/www/tutorial31/tutorial31.html

Algoritmo de DeCasteljau

- Superfícies de grau n $\rightarrow \frac{(n+1)(n+2)}{2}$ pontos



Interpolações convexas

$$b_I^r = u\, b_{I+100}^{r-1} + v\, b_{I+010}^{r-1} + w\, b_{I+001}^{r-1}$$

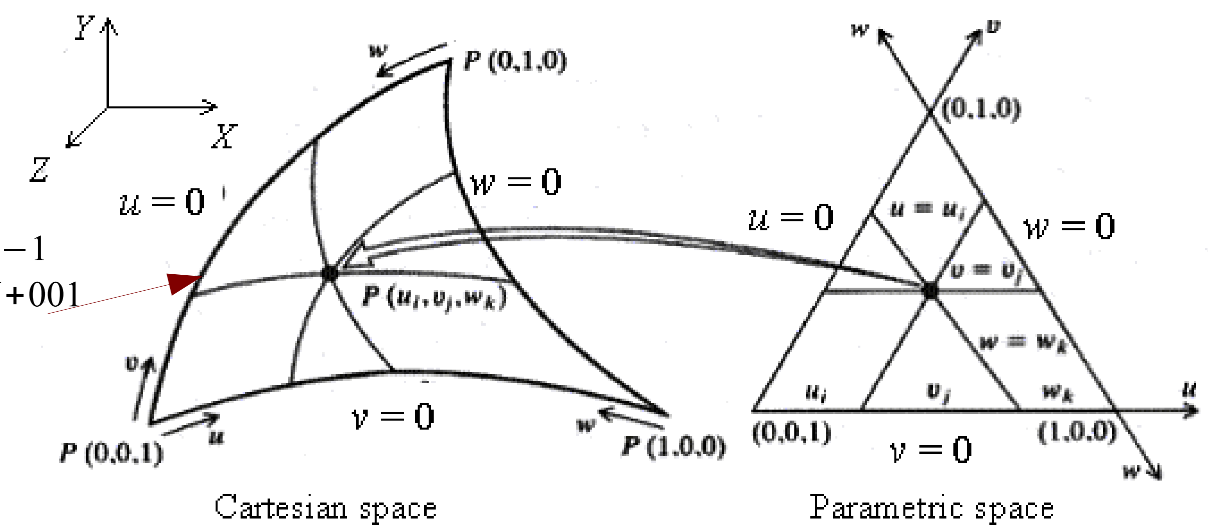
$b_{\vec{0}}^n(u, v, w)$: ponto sobre a superfície

Propriedades

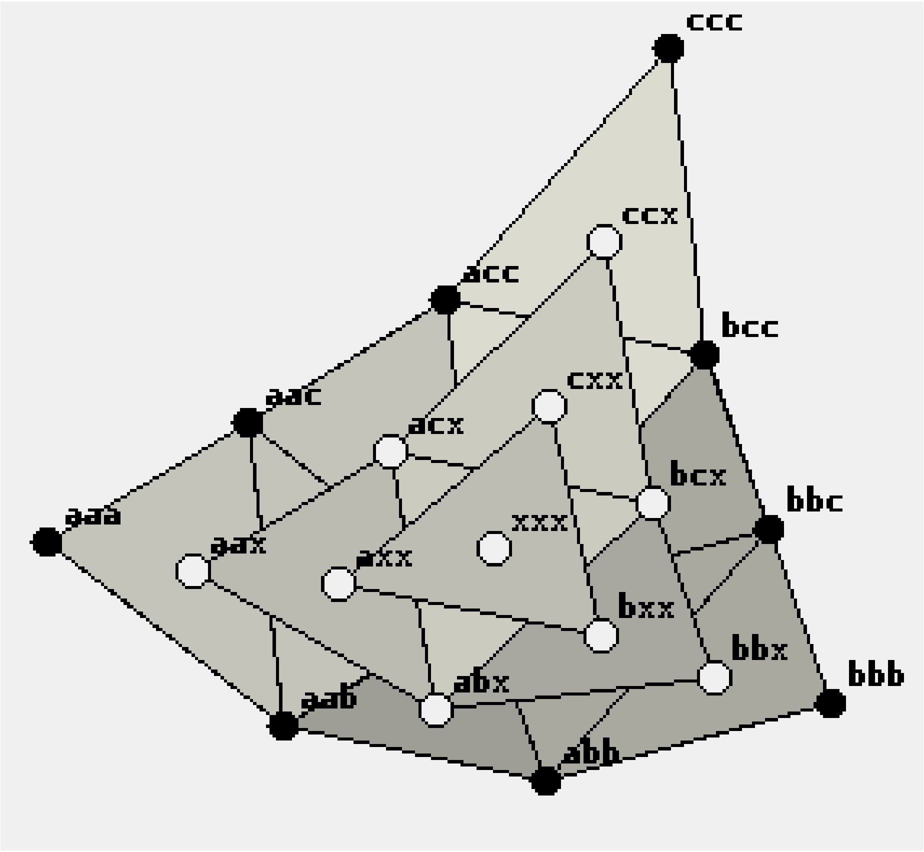
- Invariantes sob transformações afins.
- Invariantes sob transformações afins dos parâmetros.
- Convexidade
- Borda é constituída pelas curvas de Bézier

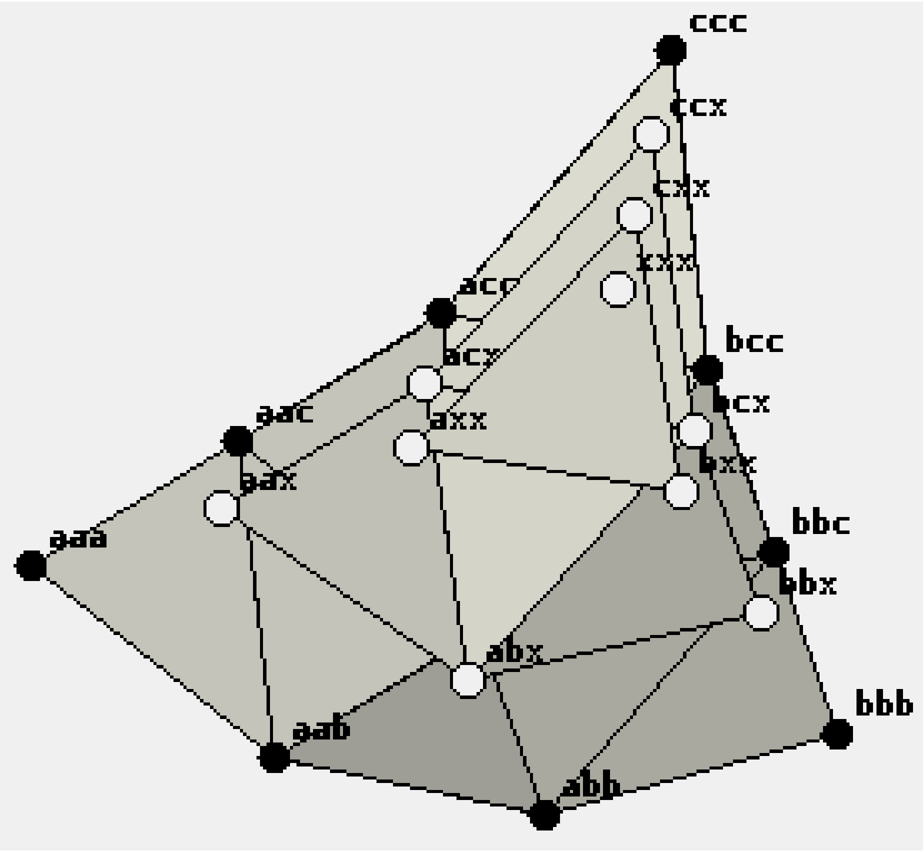
$$b_I^r(0,u,w) = u b_{I+010}^{r-1} + w\, b_{I+001}^{r-1}$$

$$u + w = 1$$



Blossom





Combinação convexa de 3 pontos

http://cagd-applets.webarchiv.kit.edu/mocca/html/noplugin/BezierTriangle/AppDeCasteljau/index.html

Blossom

$$P(s,s,s)$$
$$P(s,s,t)$$
$$P(s,s,u) \quad P(s,s,[s,t,u])$$
$$P(t,t,s)$$
$$P(t,t,t)$$
$$P(t,t,u) \quad P(t,t,[s,t,u]) \quad P(s,[s,t,u],[s,t,u])$$
$$P(u,u,s)$$
$$P(u,u,t)$$
$$P(u,u,u) \quad P(u,u,[s,t,u]) \quad P(s,u,u)$$

Funções de Bernstein

- Triângulo de Bézier de grau 3

$$P(s,t,u)=(\alpha s+\beta t+\gamma u)^3=\beta^3 t^3+3\alpha\beta^2 s t^2+3\beta^2\gamma t^2 u+3\alpha^2\beta s^2 t$$
$$6\alpha\beta\gamma stu+3\beta\gamma^2 t u^2+3\alpha^2 s^3+3\alpha^2\gamma s^2 u+3\alpha\gamma^2 s u^2+\gamma^3 u^3$$

- Triângulo de Bézier de grau n

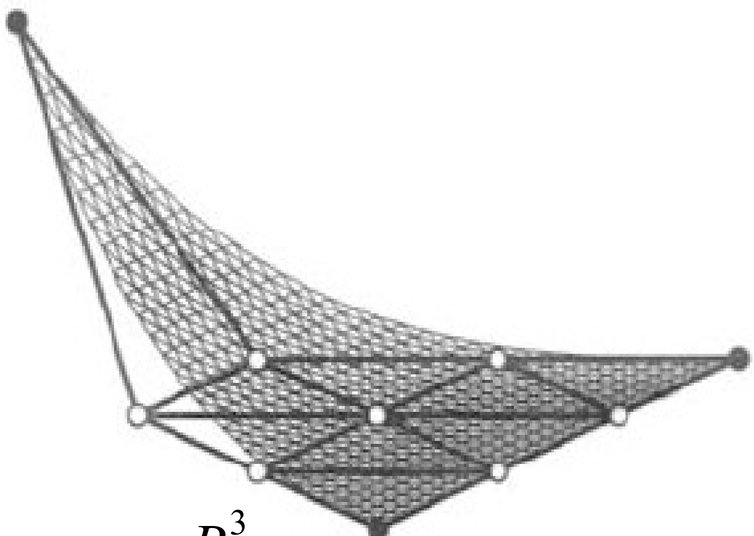
$$P(s,t,u)=(\alpha s+\beta t+\gamma u)^n=\sum_{i+j+k=n}\binom{n}{ijk} s^i t^j u^k \alpha^i \beta^j \gamma^k$$

$$\sum_{i+j+k=n}\frac{n!}{i!\,j!\,k!} s^i t^j u^k \alpha^i \beta^j \gamma^k=\sum_{i+j+k=n} b_{ijk} B^n_{ijk}(s,t,u)$$

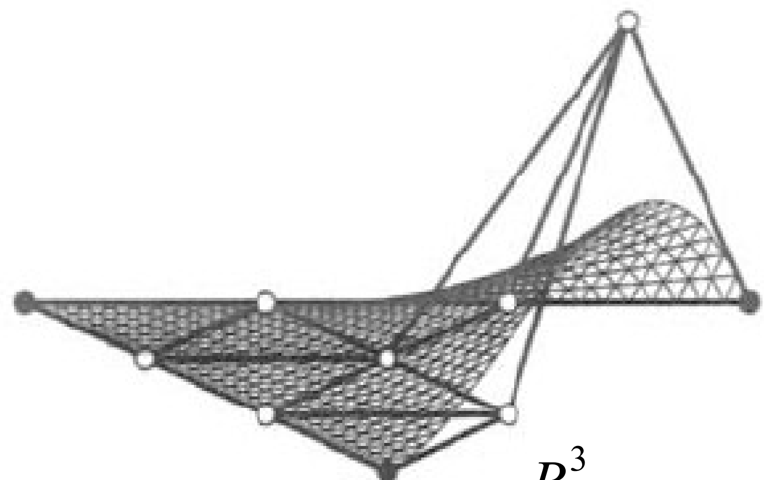
Funções de Base

Pontos de controle

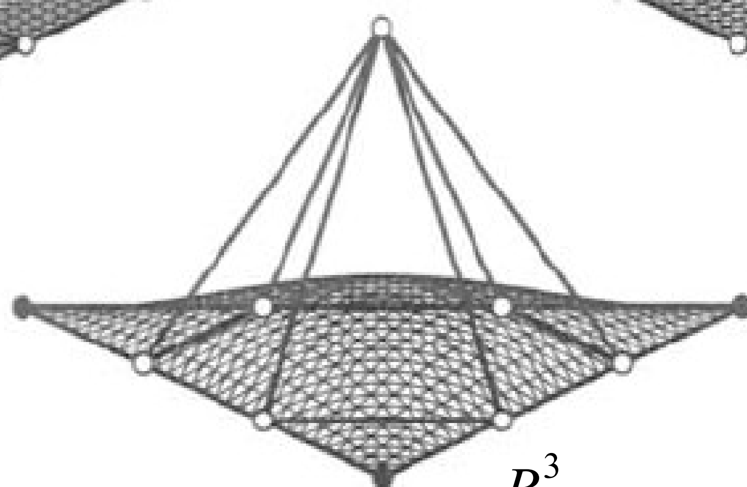
Funções de Bernstein



B^3_{003}



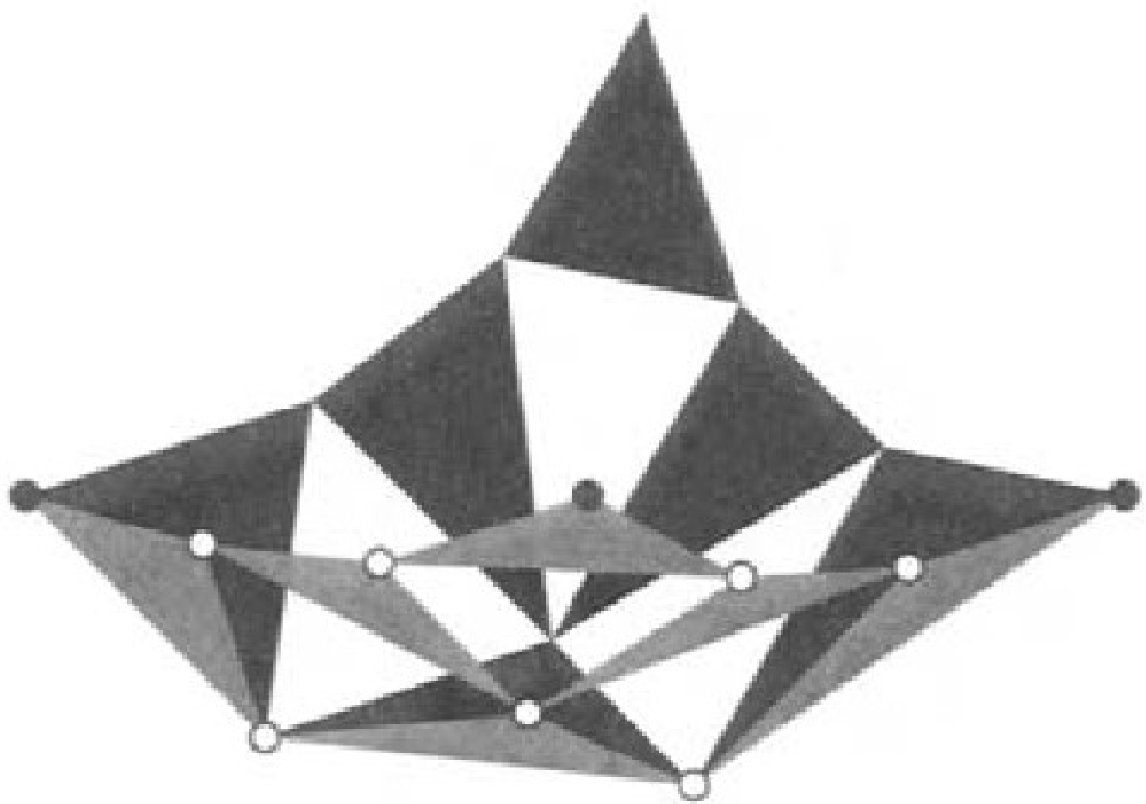
B^3_{120}



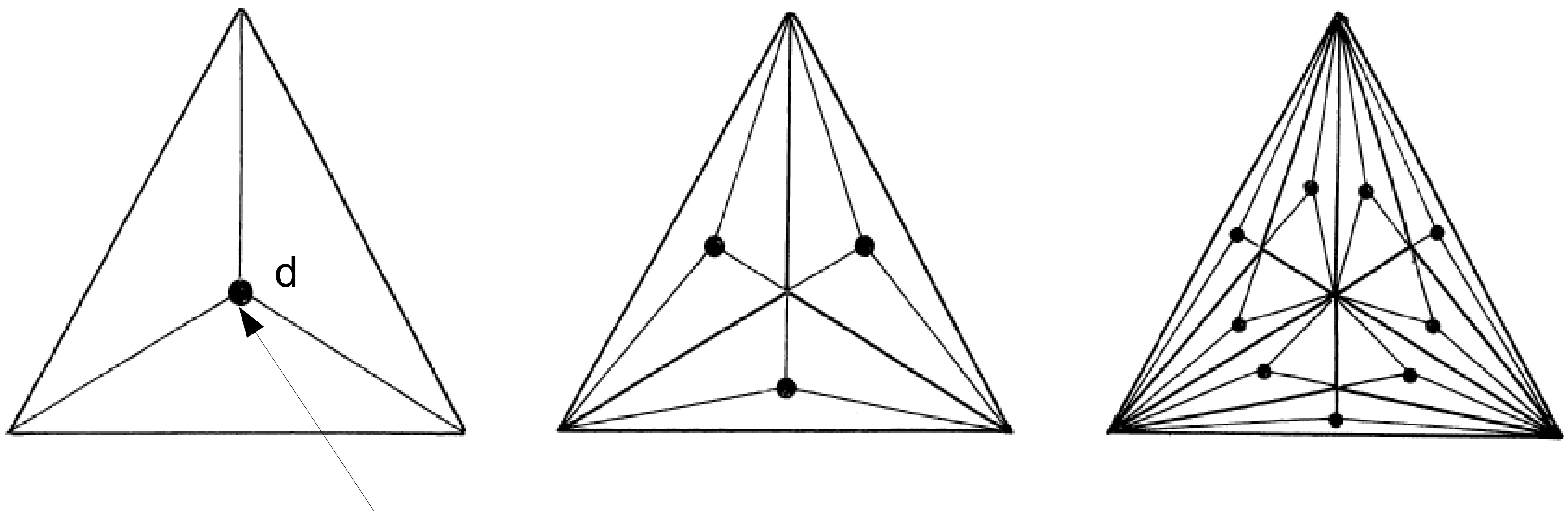
B^3_{111}

Subsivisões

•Pontos intermediários do algoritmo de de Casteljau formam pontos de controle dos sub-triângulos de Bézier



Subdivisãoes pelos baricentros



$b^n(d)$ divide o triângulo Bézier em 3 triângulos Bézier

http%3A%2F%2Fwww.springer.com%2Fcda%2Fcontent%2Fdocument%2Fcda_download
document%2F9783540437611-c1.pdf%3FSGWID%3D0-0-45-85135-p2258136&ei=Bn8Y
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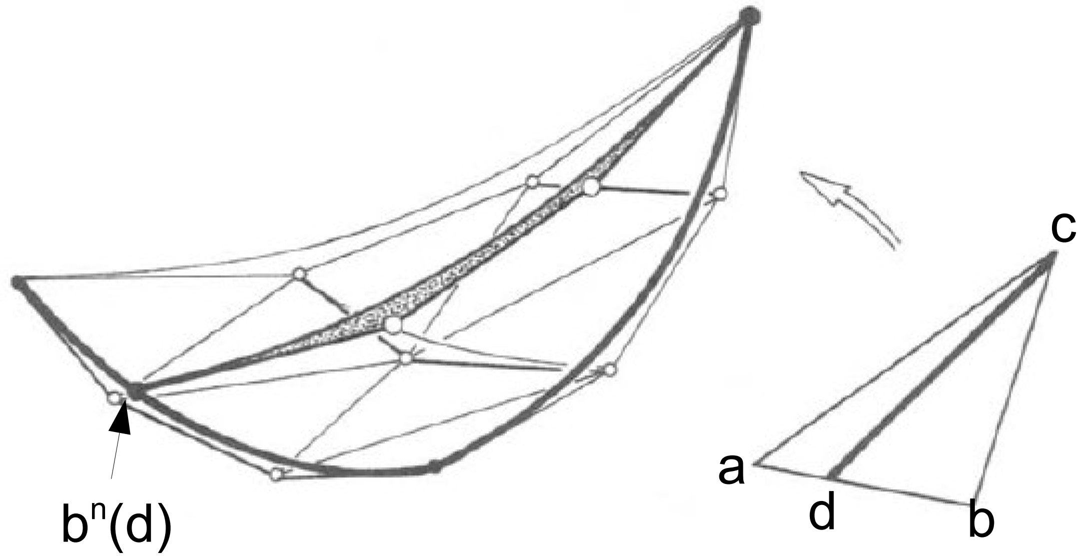
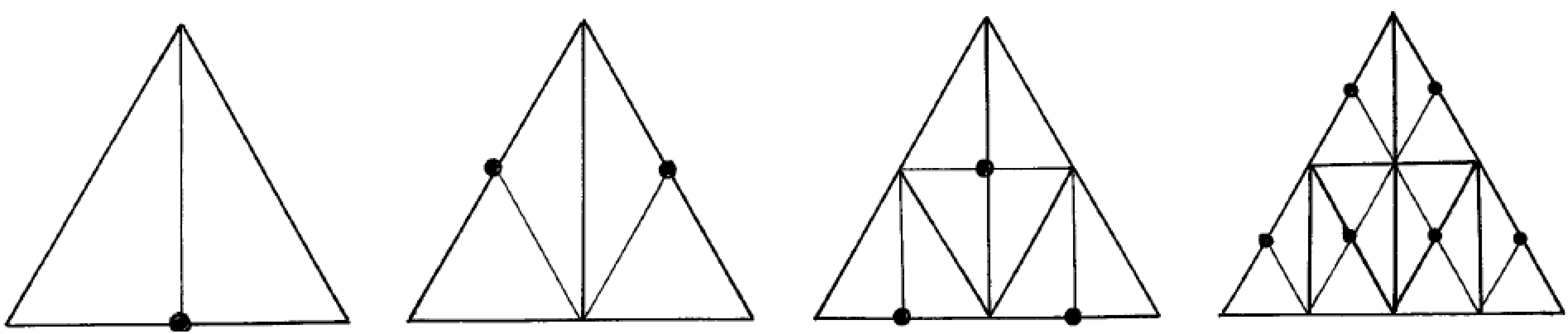
Subdivisões pelas linhas radiais





Triângulos de Bézier Racionais

$$b(u) = \frac{\sum_{I=n} w_i b_i B_i^n(u)}{\sum_{I=n} w_i B_i^n(u)}$$