# Cloth Modeling with a Discrete Cosserat Surface

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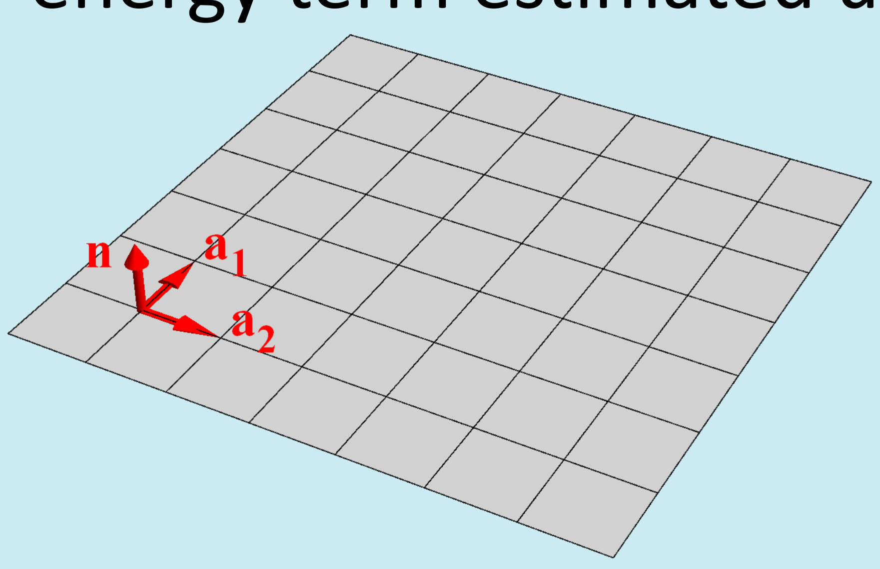
## Motivation: A sheet based non-data-driven physically-based cloth simulation model on GPU.

**Proposal:** To apply the mechanical equilibrium equation with the internal potential energy term estimated using the Cosserat surface theory [1].




Sample surface **r(u,v)** on instant $t_0$

$$\mu\frac{\partial^2 \boldsymbol{r}(t)}{\partial t^2} + \varrho\frac{\partial \boldsymbol{r}(t)}{\partial t} + \boldsymbol{K}(\boldsymbol{r},t)\boldsymbol{r}(t) = \mu\boldsymbol{F}(\boldsymbol{r},t)$$

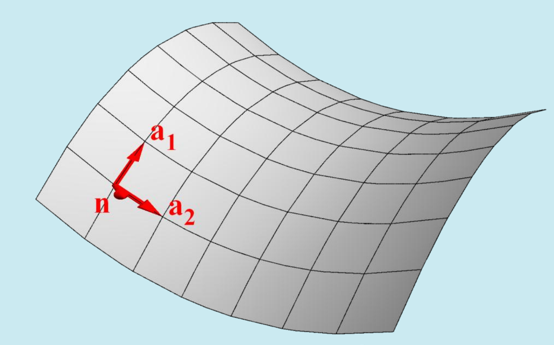
$$\boldsymbol{K}(\boldsymbol{r},t)\boldsymbol{r}(t) \approx \mu\frac{\partial\mathcal{A}(\boldsymbol{r},t)}{\partial \boldsymbol{r}(t)} = \frac{\partial\mathcal{A}(\boldsymbol{r},t)}{\partial\varepsilon(t)} + \frac{\partial\mathcal{A}(\boldsymbol{r},t)}{\partial\kappa(t)}$$

$$\mathcal{A}(\boldsymbol{r},t) = \Phi\varepsilon_{\alpha\beta}(t)\varepsilon_{\gamma\delta}(t) + \Psi\kappa_{\alpha\beta}(t)\kappa_{\gamma\delta}(t) + \Theta\varepsilon_{\alpha\beta}(t)\kappa_{\gamma\delta}(t)$$

$\Phi, \Psi, \Theta$ are material and initial state geometry dependent constants

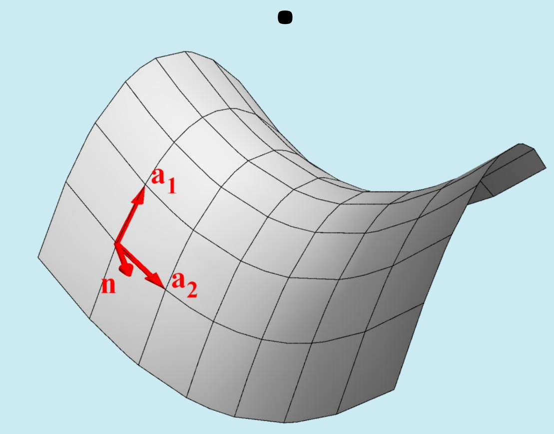
$$\kappa_{\alpha\beta}(t) = -\left(b_{\alpha\beta}(t) - b_{\alpha\beta}(t_0)\right) \qquad \varepsilon_{\alpha\beta}(t) = \frac{1}{2}\left(a_{\alpha\beta}(t) - a_{\alpha\beta}(t_0)\right)$$

$a_{\alpha\beta}$ is the metric tensor and $b_{\alpha\beta}$ is the curvature tensor




$t_1 = t_0 + \Delta t$

⋮




$t_n = t_0 + n\Delta t$

**Problems:**

- Applying the Cosserat model on meshes of arbitrary topology
- Representation of these meshes on GPU [2]
- Boundary conditions for the numerical solution
- Parallelism of numerical resolution

**Implementation:** per vertex processing

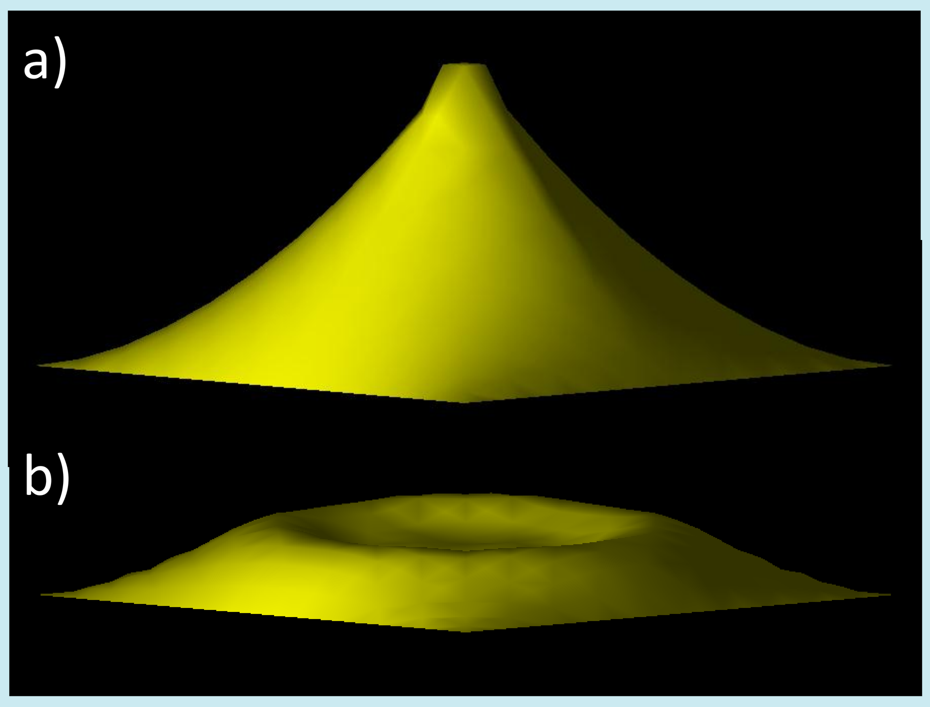
**setup**

- Calculate basis $\{\mathbf{a_1}(t_0),\mathbf{a_2}(t_0)\}$, estimate $a_{\alpha\beta}(t_0)$ and $b_{\alpha\beta}(t_0)$ and store the values

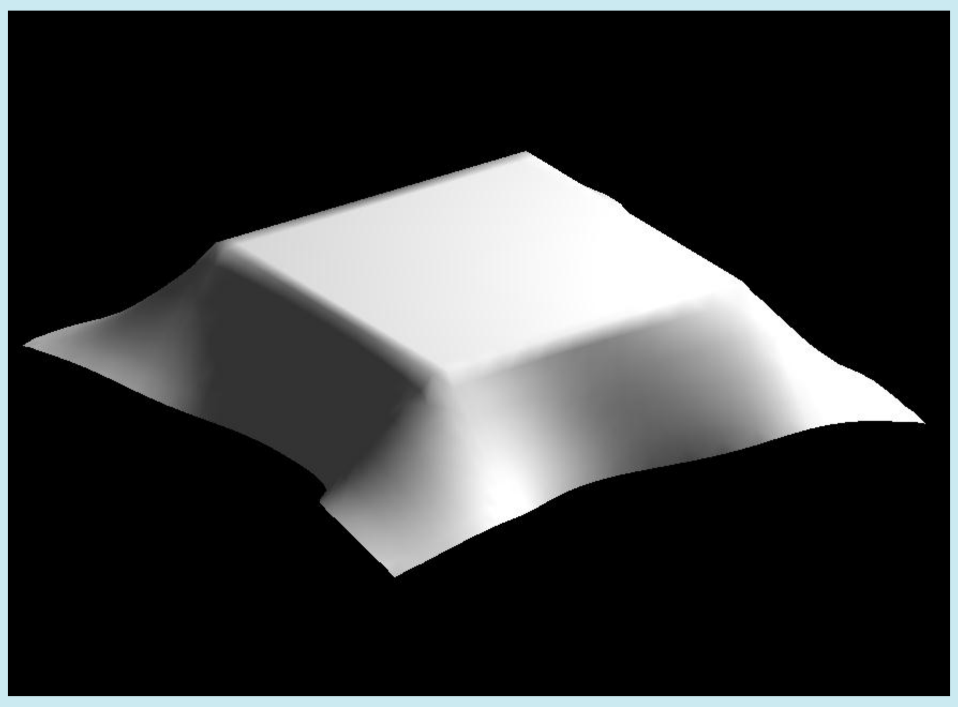
**Implemented on GPU**

- Calculate basis $\{\mathbf{a_1}(t),\mathbf{a_2}(t)\}$ → Estimate $a_{\alpha\beta}(t)$ and partial derivatives → Estimate $b_{\alpha\beta}(t)$
- → Compute $\varepsilon(\boldsymbol{r},t)$ and $\kappa(\boldsymbol{r},t)$ and calculate $\mathcal{A}(\boldsymbol{r},t)$ → Calculate internal forces → Estimate new positions → (back to Calculate basis)

**Results:** Samples of the animation of a flat square sheet and the average execution times of the metric and the curvature tensors on CPU and GPU for different mesh sizes.




elastic behavior **(a)** under a pull-up force in the middle and **(b)** after removing the force



draping behavior of a less elastic cloth fabric under gravitational force

| Mesh size | Avg CPU time (ms) | Avg GPU time w/ copy overhead (ms) | Gain |
|---|---|---|---|
| 20x40 | 1.7965 | 2.1704 | 0.8277x |
| 40x80 | 7.4238 | 5.2288 | 1.4198x |
| 80x160 | 29.7666 | 15.0957 | 1.9719x |


**References:**
[1] A. E. Green, P. Naghdi, and W. Wainwright, "A general theory of a cosserat surface", Archive for Rational Mechanics and Analysis, vol. 20, 1965.
[2] W.-W. Feng, Y. Yu, and B.-U. Kim, "A deformation transformer for real-time cloth animation", ACM Trans. Graph., vol 29, 4, 2010.


**Further information:** http://www.dca.fee.unicamp.br/projects/desmo